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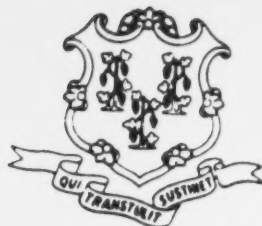
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ABSTRACT

The purpose of this project is to help teachers of mathematics use the computer to aid the teaching and learning of many mathematical concepts in a meaningful way. To this end, commercially available software packages are described which are versatile, cost effective, and easy to use. A review and several lessons, each including homework exercises, have been written for each package and are presented in ready-to-use form. Chapter 1, "The Factory," presents four lessons simulating an assembly line. Chapter 2, "Gears: Strategies in Problem Solving," contains four lessons of puzzle building and solving. Chapter 3, "Building Perspective," provides four lessons for developing visual thinking skills (spatial perception and spatial reasoning). Chapter 4, "The Geometric Supposer: Triangles," includes lessons on random points of triangles, medians of a triangle, and Heron's formula. Chapter 5, "The Geometric Supposer: Quadrilaterals," offers two lessons about areas of parallelograms and quadrilaterals. All five software packages reviewed are published by Sunburst Communications. (YP)

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A REVIEW
OF
SELECTED MICROCOMPUTER SOFTWARE PACKAGES
WITH LESSONS
FOR TEACHING MATHEMATICS
GRADES 8 - 12
VOLUME 3

A Curriculum Development Project
of the
Project to Increase Mastery of Mathematics and Science

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FOR TEACHING MATHEMATICS
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VOLUME 3

A Curriculum Development Project
of the
Project to Increase Mastery of Mathematics and Science

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APRIL 1989

HISTORY, GOALS, AND SUPPORT OF pimms

The Project to Increase Mastery of Mathematics (pimm) began in 1979 as a joint effort of a diverse group of volunteers-- individuals from education, business, government, industry, and labor-- to foster greater command, understanding, and appreciation of mathematics by graduates of Connecticut schools. The symbiotic relationship of mathematics and its applications was a pervasive theme of the various pimm undertakings.

In 1983, the organization broadened its scope to become the Project to Increase Mastery of Mathematics and Science (pimms), and at the same time expanded its activities by mounting a Fellowship Program: notably able Connecticut high school teachers of mathematics and science were invited to study together at Wesleyan in courses and seminars as a prelude to offering consultative assistance on curricular matters to schools requesting such service and to providing workshops to teachers for their professional development.

Several groups of Fellows are devoting the bulk of their energies to writing projects. The materials they prepare will be broadly disseminated to facilitate the introduction into school curricula of mathematical applications of major importance and considerable attractiveness.

Now several Fellowship Programs are in operation, extending the concept to middle school and elementary school teachers. A significant value of the Fellowship Programs is found in the colleagueship developed among professionals working in different institutions at varied levels.

Throughout its growth, pimms has continued as a partnership of individuals and organizations. The list of devoted volunteers is too long to be recorded here. Major financial support of pimms has been supplied by the Connecticut Department of Education, the Connecticut Department of Higher Education, the Charles E. Culpeper Foundation, the General Electric Foundation, GTE, Divisions of United Technologies Corporation (Hamilton Standard, Norden Systems, Otis Elevator, Pratt and Whitney Manufacturing, Research Center, Sikorsky Aircraft), and Wesleyan University. Substantial contributions have also been made by Amax Foundation, Army Research Office, Boehringer Ingelheim, Chesebrough-Ponds, Choate Rosemary Hall, CIGNA, Connecticut Academy of Science and Engineering, General Signal, the Edward W. Hazen Foundation, Northeast Utilities, Omega Group, The Perkin-Elmer Corporation, Pfizer Inc., Pitney Bowes, Raymond Engineering, SNET, Stanley Works, Union Carbide Corporation, and Xerox Corporation.

All of us who are concerned for high quality education owe a great debt of gratitude to these organizations for their corporate citizenship.

R. A. Rosenbaum
Director, pimms

FOREWORD

In the report, How Schools Use Microcomputers, Henry Jay Becker stated, "... with an organized and sustained strategy for using computers to improve the effectiveness of school instruction we might accomplish what has eluded education throughout recent decades -- clear and obvious improvements in student accomplishment and in how much learning occurs in a day in school. Nothing would help the institution or its practitioners more." *

The preparation of this third volume is a step towards the realization of that goal.

Connecticut school districts may contact the **pimms** office for information on scheduling an Awareness Workshop or other in-service program related to the use of computers for mathematics instruction.

We earnestly solicit the comments of users of this material. Please send your suggestions for improvements, reviews, or lessons to:

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Mathematics Coordinator, **pimms**
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Middletown, CT 06457

*Becker, Henry Jay, How Schools Use Microcomputers: Summary of the 1983 National Survey, The Johns Hopkins University Center of Social Organization of Schools; Baltimore, 1985.

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INTRODUCTION

In the late '70s and early '80s the microcomputer burst upon the scene of public education. School districts purchased the hardware and used the equipment mainly to teach various programming languages. While the potential power of this new technology was recognized, very few school systems developed programs to incorporate the use of the computer as an instructional tool.

The purpose of this project is to help teachers of mathematics get started using the computer to aid the teaching and learning of many mathematical concepts **in a meaningful way**. To this end, the writing team has chosen commercially available software packages which are versatile, cost effective, and easy to use. A review and several lessons, each including homework exercises, have been written for each package and are presented in ready-to-use form. The material may be photocopied for classroom use.

The lessons presented in this volume are designed to promote interaction between the teacher and the students and among the students. Many lessons are developmental and should be followed by a more formal treatment of the concepts involved. Contrary to popular belief, this type of activity does not increase the time needed to teach the concepts but improves efficiency and effectiveness by promoting the understanding of a problem, the ability to visualize the problem, and the ability to verbalize a statement of the solution of the problem.

We hope that this **third volume** will help to spark the imagination of creative teachers who will then begin to develop their own lessons. Although all of the software reviewed herein relates to the Apple II microcomputer family, some of the packages may be available for other computers. Users are advised to check software catalogs for such availability. As more and better instructional software becomes available for the Macintosh and IBM microcomputers, reviews and lessons for them will be included in future editions of this publication.

It should be noted that software packages that already contain excellent instructional support material -- such as the four volumes of Compter Graphing Experiments (Addison-Wesley) and Mathematics: Volume 1, distributed by the Minnesota Educational Computing Consortium (MECC)-- have not been included here; however, any school district interested in developing a core library of software for mathematics instruction should include those packages with their initial purchases.

Also, you are advised to read carefully the restrictions on use of software to assure that your use does not violate copyright regulations. Many software packages are available in laboratory sets so that a sufficient number of diskettes will be available for a laboratory activity. (Users also should be aware that even when the booting of multiple machines is allowed by the copyright rules, it is an inefficient and inconvenient practice.)

COMPUTER SOFTWARE PROGRAM REVIEW AND DESCRIPTION

TITLE: THE FACTORY

PUBLISHER: Sunburst Communications
39 Washington Ave.
Pleasantville, N.Y. 10570

PRICE: \$65.00
10 - disk lab pack \$195.00

COMPUTERS: Apple II, II+, IIc, IIe, Atari, Acorn,
Commodore 64, IBM PC/PCjr, Tandy 1000,
TRS-80 Color

PIMMS RATING: Very Good

PROGRAM APPROPRIATE FOR:

Courses: 7th and 8th grade math, General Math,
Geometry, problem-solving classes

Ability Level: Elementary school: high ability
Middle school: average/high
High school: low/average/high

Format: instructional, discovery/laboratory,
drill and practice

Characteristics: good sound, good animation graphics,
disk-dependent, user friendly

ADDITIONAL PERIPHERALS NEEDED: color monitor preferred

REVIEW AND DESCRIPTION OF THE PROGRAM:

The Factory is a program whose operation permits simulation of an assembly line (the factory). A square piece of blank stock is sent through the line. The user then creates a product via the selective use of three machines -- a hole-punching machine (punches up to 3 square or round holes), a striping machine (paints up to 3 thin, medium, or wide stripes, and a machine that rotates the product through up to four 45-degree counterclockwise rotations. The assembly line may contain up to 8 stations.

The Main Menu offers three activities:

1. Testing a Machine: Further options are offered - hole punching (shape and number), rotation in degrees (45,90,135,180), and striping (three widths).
2. Building a Factory: The user sets up an assembly line involving up to 8 stations and can watch the production of the product. A program option permits the user to challenge a friend to reproduce the product.
3. Making a Product: The user is challenged to build a factory (design the sequence of stations in the line) which will duplicate a product shown on the screen. Three levels of difficulty are available.

Product construction varies from easy (requiring one or two steps) to difficult (requiring up to eight steps). There is no limit to the number of times the user may attempt reconstruction. This feature encourages the young "engineers" to design and modify strategies, both individually and in groups.

Students at various age levels can derive satisfaction from recognizing design elements and predicting product configurations and will delight in devising elaborate products and challenging their friends to reproduce them.

The program supports several problem-solving strategies: analyzing a process, determining sequence, and working backwards.

The program can be used with a variety of ability levels from grades 4 to 12. It is appropriate for a variety of curriculum units and is adaptable to individual or group instruction.

GENERAL INFORMATION FOR BEGINNING USERS
STARTING THE PROGRAM USING AN APPLE II COMPUTER

STARTING THE PROGRAM

1. Grasp the disk at the top (label area) between your thumb and index finger. Carefully remove the computer diskette from its envelope.
2. Open the door of the disk drive. Insert the program diskette into the disk drive with the label to the right and facing up. Then close the door of the disk drive.
3. Turn on the monitor. (Ask your instructor for directions. Different types of monitors may be used with Apple computers.)
4. Turn on the Apple IIe computer. (The switch is at the left side of the computer, on the back panel.)
5. A red light on the disk drive will come on. If the disk light does not go off in about 10 seconds, turn the computer off and make sure your diskette is placed correctly in the disk drive.
6. "SUNBURST" will appear on the screen, followed by the Main Menu.
7. Proceed to the lesson.

TURNING OFF THE SYSTEM

1. Return to the Main Menu.
2. Remove the diskette from the disk drive, put it back in its envelope, and return it to its storage place.
3. Turn off the Apple monitor; turn off the computer.

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: TESTING A MACHINE: Punching and Striping

OBJECTIVES:

- to gain practice with the punching and striping operations
- to recognize the commutativity of these operations

COURSES: General Math, Career Math, or Applied Math

SOFTWARE: THE FACTORY

TIME: one class period

OTHER MATERIALS: overhead projector

PREREQUISITE SKILLS/CONCEPTS:

- recognition of squares and circles
- understanding of the concept of commutativity

FORMAT: discovery lab

GENERAL DESCRIPTION OF LAB ACTIVITY:

After a brief demonstration/overview of the Description menu, the teacher should present (via transparency diagrams) a step-by-step look at the punching and striping operations, both singly and in various combinations.

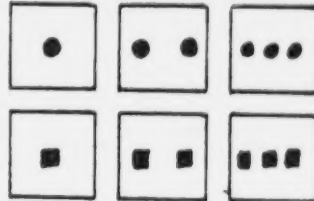
The students then may use the program to practice these operations. The Laboratory Exercises provide for making predictions and testing outcomes using the operations singly and in combinations. Some of the exercises provide for student testing of the reversibility of certain punching and striping operations.

TESTING A MACHINE: PUNCHING AND STRIPING
LABORATORY EXERCISES
TEACHER'S GUIDE

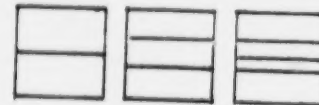
TEACHER/STUDENT DIALOGUE:

- 1) The teacher should introduce the lesson via the Description Menu choice to illustrate making menu choices and the availability of the operations. Next, a series of transparency diagrams may be used to show the effects of the punching and striping operations.
(Rotation will be addressed in the next lesson.)

- 2) Punching:



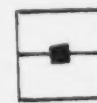
- Striping:



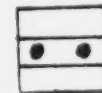
Lab exercises 1 and 2 will provide practice.

- 3) Next, illustrate combinations of punching and striping.

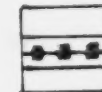
punch - square - one; stripe - thin



punch - circle - two; stripe - medium



punch - square - three; stripe - thick



Lab exercise 3 will provide practice.

- 4) Finally, several examples of the commutativity of these operations should be illustrated: stripe - punch and punch - stripe.

Lab exercise 3 will provide practice.

Getting Started:

Select #1: Testing a Machine.

The next screen will ask: Do you want a circle or a square?
Select square. (RETURN)

The next screen will ask: How many punches: one,two,three
Select one. Before you hit RETURN, draw your prediction for
the appearance of the product.

Predictions:	Screen product:	P
--------------	-----------------	---

The next screen will ask: Do you want to make another product? yes no Select yes. (RETURN)

Use the menu selection technique given above. Choose the stripe machine to place a thin stripe across the square. Predict the appearance of your product.

Prediction: Screen product: S

The following laboratory exercise will give you practice in using the menu selection techniques given above.

Step	Machine	Selections	Prediction:
#1	punch	circle two	A
			Right? Y N

Step	Machine	Selections	Prediction:
#1	punch	square three	B
			Right? Y N

EXERCISE 2 The Stripe Machine

Step	Machine	Selections	Prediction:
#1	stripe	medium	A
			Right? Y N
Step	Machine	Selections	Prediction:
#1	stripe	thick	B
			Right? Y N

Now let's learn to combine the punching and striping operations. To do this, go back to the Main Menu and select #3: Build a Factory.

The next screen will show: Select a machine: punch, rotate, or stripe. Select punch. (RETURN)

The next screen asks: What kind of punch? square or circle. Select square. (RETURN)

The next screen will ask: How many punches?: one, two, three. Select one. Before you hit Return, draw your prediction for the appearance of the first machine.

Prediction:

PS

After your prediction is recorded, hit Return and the screen will again offer: Select a machine. This time select stripe. (RETURN)

The next screen will ask: What size stripe do you want: thin, medium, or thick? Select thick. (RETURN)

Draw your prediction of the effect of following the punching operation with the striping operation.

Prediction:

ST

The next screen will show: Select a machine: punch, rotate, stripe, done, or erase. Select done. (RETURN)

EXERCISE 3 Combination of Punching and Striping

Step	Machine	Selections	Prediction:
#1	punch	circle two	A
#2	stripe	medium	
			Right? Y N
Step	Machine	Selections	Prediction:
#1	stripe	medium	B
#2	punch	circle two	
			Right? Y N

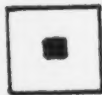
EXERCISE 4 Design Your Own Product

Use the punch and stripe machines. Design three products of your own.

List here the menu choices you'll make	Draw a prediction of the appearance of your product	Copy here the product shown on the screen
Product A		
Product B		
Product C		

TESTING A MACHINE: PUNCHING AND STRIPING
ANSWERS TO LABORATORY EXERCISES

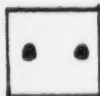
P.



S.



1A.



1B.



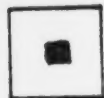
2A.



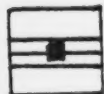
2B.



PS



ST



3A.



3B.



4. ANSWERS MAY VARY.

TESTING A MACHINE: PUNCHING AND STRIPING HOMEWORK EXERCISES

1. Use the information in the chart. Draw your prediction for the appearance of the product. (You can test your predictions via the computer in the next class period.)

Step	Machine	Selections	
#1	punch	square	three

Prediction:

A

Right? Y N

Step	Machine	Selections	
#1	stripe	medium	

Prediction:

B

Right? Y N

Step	Machine	Selections	
#1	stripe	thick	
#2	punch	circle	two

Prediction:

C

Right? Y N

Step	Machine	Selections	
#1	punch	square	one
#2	punch	circle	two

Prediction:

D

Right? Y N

2. Create your own two-step product. (Try to make it different from those you saw in class.)

Step	Machine	Selections	
#1			
#2			

Prediction:

A

Right? Y N

Step	Machine	Selections
#1		
#2		

Prediction:

B

Right? Y N

Step	Machine	Selections
#1		
#2		

Prediction:

C

Right? Y N

Step	Machine	Selections
#1		
#2		

Prediction:

D

Right? Y N

3. Use a punch machine and a stripe machine. Design a product that can be made in two reversible steps. Show both methods.

Method 1

Step	Machine	Selections
#1		
#2		

Prediction:

A

Right? Y N

Method 2

Step	Machine	Selections
#1		
#2		

Prediction:

B

Right? Y N

TESTING A MACHINE
PUNCHING AND STRIPING
ANSWERS TO HOMEWORK EXERCISES

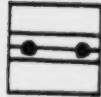
1) A.



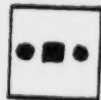
B.



C.



D.



Responses to Exs. 2 through 4 will vary.

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: TESTING A MACHINE: Rotation

OBJECTIVE:

To gain practice with and understanding of the effects of using the rotation options in the program.

COURSES: General Math, Career Math, Applied Math

SOFTWARE: THE FACTORY

TIME: one class period

OTHER MATERIALS: overhead projector

PREREQUISITE SKILLS/CONCEPTS:

- recognition of geometric shapes (squares, circles)
- familiarity with the use of a 180-degree protractor
- know meaning of counterclockwise (...and clockwise...)

FORMAT: discovery lab

GENERAL DESCRIPTION OF LAB ACTIVITY:

The Testing a Machine option, may be used to show the menu choices needed to create a product under the rotation option.

After giving a brief demo of making menu choices, the teacher may wish to examine more closely the concept of rotation (in steps of 45 degrees) via a series of transparency diagrams.



Students then may be directed to illustrate the rotation by using pieces of appropriately marked cardboard.



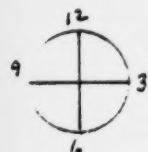
The lab exercises involve rotation, rotation with striping, rotation with punching, rotation with striping and punching, as well as examination of the reversibility (commutativity?) of the rotate-stripe and rotate-punch operations.

TESTING A MACHINE: ROTATION
LABORATORY EXERCISES
TEACHER'S GUIDE

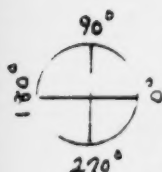
1. Ask the students to picture a one-handed clock and how such a clock would appear if time were counted backwards from 3 p.m. to 3 a.m...



2. Keeping the clock in mind, have the students partition the 12-hour clock into quarters, and then ask, "How many hours are there in each quarter?" (3)



3. Show a circle (360 degrees) and then the relationship between 3-, 12-, 9-, 6-, and 3 o'clock with the degrees of a circle -- i.e., 3 o'clock marks either 0 or 360 degrees, 12 o'clock marks 90 degrees, 9 o'clock marks 180 degrees, etc.



4. Next, ask the students to divide each quarter in half and then find the number of degrees for each new part. [Compare the foregoing activity with cutting a pizza into 8 slices.]



5. Model this activity using cardboard pieces, chalkboard, or overhead projector: Start at the 3 o'clock position and show four successive 45-degree stepwise counterclockwise rotations -- i.e., $0 + 45 = 45$, $45 + 45 = 90$, $45 + 45 + 45 = 135$, $45 + 45 + 45 + 45 = 180$.

6. Use the marked circle on the next page. Show how the arrows inside each square match up with the arrows that mark the degrees as the square is rotated around the center from 0 to 180 degrees. Also point out how the orientation of the square changes as the square rotates.

7. Now direct the students to hold up their cardboard shapes to represent

a 45 degree rotation

a 90 degree rotation



Continue with this activity until students understand the stepwise counterclockwise rotation process.

7. Note: Be sure to model combinations of processes before they are encountered in Laboratory Exercise 2.

TESTING A MACHINE: ROTATION
LABORATORY EXERCISES

Making Menu selections:

Under the direction of your teacher load and boot The Factory.
The Main Menu will appear. Select #1: Testing a Machine.

The next screen will offer a further choice: punch, rotate,
stripe. Select rotate. (RETURN)

The next screen will ask: How many degrees do you want:
45, 90, 135, or 180? Select 90. Before you hit Return,
draw here your prediction of the appearance of the product.

Prediction:

Screen product:

The next screen will ask: Do you want to make another product?
yes - no Select yes (RETURN)

Use the above menu selection techniques as you continue with
the lab exercises.

EXERCISE 1 - Rotation

Step	Machine	Selections
#1	rotate	45

Prediction:

A

Right? Y N

Step	Machine	Selections
#1	rotate	135

Prediction:

B

Right? Y N

EXERCISE 2 Rotation with Striping

To combine machines, go back to the Main Menu and select #3: Build a Factory.

The next screen will state: Select a machine: punch, rotate, or stripe. Select stripe. (RETURN)

The next screen asks: What size stripe: thin, medium, or thick? Select thin. Before you hit Return, draw your prediction for the appearance of the first machine product.

Prediction:

After making your prediction, hit Return and the screen will offer: Select a machine. This time select rotate. (RETURN)

The next screen will ask: How many degrees do you want: 45, 90, 135, or 180? Select 45. (RETURN)

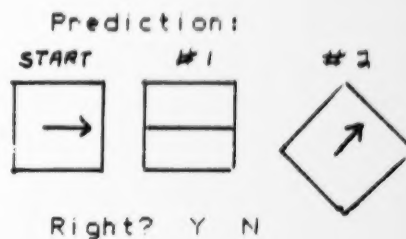
Draw your prediction for the product that results from following the 45-degree rotation with the striping.

Prediction:

The next screen will offer: Select a machine: punch, rotate, stripe, done, erase. Select done. (RETURN) Check your prediction with the screen product.

Example

Step	Machine	Selections
#1	stripe	thin
#2	rotate	45



Step	Machine	Selections
#1	stripe	thick
#2	rotate	90

Prediction:

A

Right? Y N

Step	Machine	Selections
#1	rotate	180
#2	stripe	medium

Prediction:

B

Right? Y N

Step	Machine	Selections
#1	stripe	thin
#2	rotate	135

Prediction:

C

Right? Y N

EXERCISE 3

Determine whether the next two manufacturing sequences give the same product.

Step	Machine	Selections
#1	rotate	135
#2	stripe	medium

Prediction:

A

Right? Y N

Step	Machine	Selections
#1	stripe	medium
#2	rotate	135

Prediction:

B

Right? Y N

C) Product A (is, is not) the same as Product B.

EXERCISE 4

Make two products which involve reversing the order of steps #1 and #2 as in Ex. 3. Record your selections and predictions. Check your predictions by building the products via the program.

Step	Machine	Selections	Prediction:
#1			A
#2			
			Right? Y N

Step	Machine	Selections	Prediction:
#1			B
#2			
			Right? Y N

EXERCISE 5 Rotation with punch
Predict, construct, check the product.

Step	Machine	Selections	Prediction:
#1	punch	square one	A
#2	rotate	90	
			Right? Y N

Step	Machine	Selections	Prediction:
#1	rotate	135	B
#2	punch	circle two	
			Right? Y N

EXERCISE 6

Predict, construct, check the product.

Step	Machine	Selections
#1	rotate	135
#2	punch	square three

Prediction:

A

Right? Y N

Step	Machine	Selections
#1	punch	square three
#2	rotate	135

Prediction:

B

Right? Y N

C) Does reversing the order of the steps give the same product?
Explain.

EXERCISE 7 Rotation with stripe AND punch
Predict, construct, and check the product.

Example

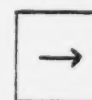
Step	Machine	Selections
#1	punch	circle one
#2	rotate	45
#3	stripe	medium

Right? Y N

Step	Machine	Selections
#1	stripe	thin
#2	punch	square two
#3	rotate	180

Prediction:

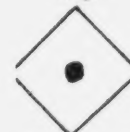
START



#1



#2



#3



Prediction:

A

Right? Y N

Step	Machine	Selections
#1	rotate	135
#2	stripe	thick
#3	punch	circle three

Prediction:

B

Right? Y N

EXERCISE 8

Predict, construct, check.

Step	Machine	Selections
#1	stripe	thick
#2	rotate	45
#3	punch	square three

Prediction:

A

Right? Y N

Step	Machine	Selections
#1	punch	square three
#2	rotate	45
#3	stripe	thick

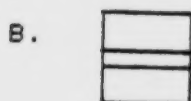
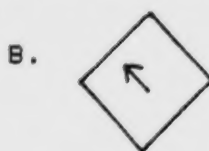
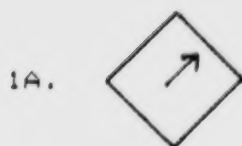
Prediction:

B

Right? Y N

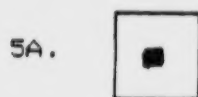
- C) Note that the steps in parts A and B are reversed .
Write a statement about whether reversing the order
of the steps ever can give the same product. Explain.

TESTING A MACHINE: ROTATION
ANSWERS TO LABORATORY EXERCISES

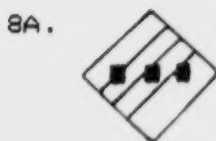


C. *is not*

4A. *Answers may vary . . . B. . . .*



C. *No*



C.

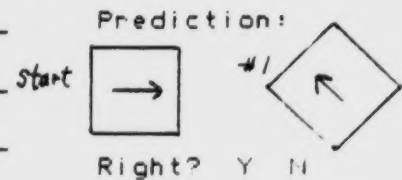
TESTING A MACHINE: ROTATION HOMEWORK EXERCISES

Design a product according to directions and predict the outcome. (Note: Vary your choices.)

1. Use one rotation operation.

Example

Step	Machine	Selections
#1	rotate	135



Step	Machine	Selections
#1		

Prediction:

A

Right? Y N

Step	Machine	Selections
#1		

Prediction:

B

Right? Y N

Step	Machine	Selections
#1		

Prediction:

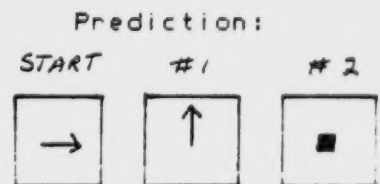
C

Right? Y N

2. Use one rotation operation and one punch operation.

Example

Step	Machine	Selections
#1	rotate	90
#2	punch	square one



Right? Y N

Prediction:

A

Step	Machine	Selections
#1		
#2		

Right? Y N

Step	Machine	Selections
#1		
#2		

Predictions:

B

Right? Y N

3. Use a rotation operation and a stripe operation.

Example

Step	Machine	Selections
#1	rotate	135
#2	stripe	thick

Predictions:

START



Right? Y N

Predictions:

Step	Machine	Selections
#1		
#2		

A

Right? Y N

Predictions:

Step	Machine	Selections
#1		
#2		

B

Right? Y N

Predictions:

Step	Machine	Selections
#1		
#2		

C

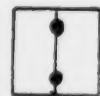
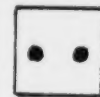
Right? Y N

4. Use combinations of the rotation, punch, and stripe operations.

Example

Step	Machine	Selections
#1	punch	circle two
#2	stripe	thin
#3	rotate	90
Right? Y N		

Prediction:



Prediction:

Step	Machine	Selections
#1		
#2		
#3		

A

Right? Y N

Prediction:

Step	Machine	Selections
#1		
#2		
#3		

B

Right? Y N

Prediction:

Step	Machine	Selections
#1		
#2		
#3		

C

Right? Y N

Prediction:

Step	Machine	Selections
#1		
#2		
#3		

D

Right? Y N

EXERCISE 5

Design a product that requires two machines. Choose your steps and selections so that when you reverse the order of the steps you DO NOT get the same product.

Step	Machine	Selections
#1		
#2		
#3		

Prediction:

A

Right? Y N

Step	Machine	Selections
#1		
#2		
#3		

Prediction:

B

Right? Y N

EXERCISE 6

Design a product that requires two machines. Choose your steps and selections so that when you reverse the order of the steps you DO get the same product.

Step	Machine	Selections
#1		
#2		
#3		

Prediction:

A

Right? Y N

Step	Machine	Selections
#1		
#2		
#3		

Prediction:

B

Right? Y N

EXERCISE 7

Design the step(s) that will produce the given product.

A.



Step	Machine	Selections
#1		

B.



Step	Machine	Selections
#1		

C.



Step	Machine	Selections
#1		

D.



Step	Machine	Selections
#1		
#2		

E.



Step	Machine	Selections
#1		
#2		

F.



Step	Machine	Selections
#1		
#2		

G.



Step	Machine	Selections
#1		
#2		
#3		

TESTING A MACHINE: ROTATION
ANSWERS TO HOMEWORK EXERCISES

Ex. 7

A.

MACHINE	SELECTIONS		
#1	ROTATE	135	

B.

MACHINE	SELECTIONS		
#1	STRIPE	MEDIUM	

C.

MACHINE	SELECTIONS		
#1	PUNCH	CIRCLE	TWO

D.

MACHINE	SELECTIONS		
#1	PUNCH	SQUARE	ONE
#2	ROTATE	45	

E.

MACHINE	SELECTIONS		
#1	ROTATE	180	
#2	STRIPE	THICK	

F.

MACHINE	SELECTIONS		
#1	PUNCH	CIRCLE	THREE
#2	STRIPE	THIN	

G.

MACHINE	SELECTIONS		
#1	STRIPE	MEDIUM	
#2	ROTATE	135	
#3	PUNCH	SQUARE	TWO

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: BUILD A FACTORY

OBJECTIVES:

- to gain practice with combining the punching, striping, and rotation operations
- to use more than one machine (with available options) in an assembly line to build a product
- to develop skills in sequencing and working backwards

COURSES: General Math, Career Math, Applied Math

LEVEL: low

SOFTWARE: THE FACTORY

TIME: one class period

PREREQUISITE SKILLS/CONCEPTS:

- familiarity with clockwise and counterclockwise rotation

FORMAT: laboratory

GENERAL DESCRIPTION OF THE LAB ACTIVITY:

The program option Build a Factory is offered in the Main Menu of the software package. Under this option the program simulates a factory assembly line containing from one to eight machines (with their options). The object is to produce a product having a specified design. A further program option permits the user to challenge a partner to reproduce the product.

BUILD A FACTORY LABORATORY EXERCISES

Making Menu Selections

Under the direction of your teacher, load and boot The Factory program. The Main Menu will appear. Select #2: Build a Factory.

The next screen will show an empty 8-station assembly line into which you will place those machine operations that you select to build your product. The screen offers: Select a machine: punch, rotate, stripe. Select punch. RETURN

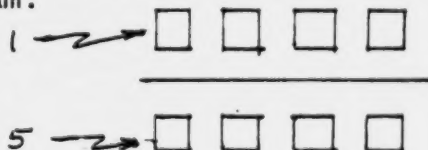
The next screen asks: What kind of punch do you want? square or circle Select circle. RETURN

The next screen asks: How many punches one, two, three? Select two. RETURN

You now have designed the first station in your assembly line. Further choices remain:

Should you want to stop building your assembly line, select done. RETURN and your product will be produced. Should you want to erase a machine, select erase. RETURN will give your previous machine.

The next screen will show: Select a machine. Select done, but before you RETURN, sketch in the space below your prediction for the product output of Station 1 in the appropriate place in the following diagram.



The next screen to appear will ask: Do you want to challenge someone? (yes or no) For now, select no. RETURN

EXERCISE 1

Predict the product of the given assembly line.

Assembly line A	Machine	Selections
Station #1	punch	square 1
Station #2	rotate	135
Station #3	stripe	thick

Prediction:

A

Right? Y N

Assembly line B	Machine	Selections
Station #1	rotate	180
Station #2	stripe	medium
Station #3	rotate	90
Station #4	punch	circle 2

Prediction:

B

Right? Y N

EXERCISE 2

Reverse the order of the machines in Assembly line A.
Will the same product be produced?

Assembly line AR	Machine	Selections
Station #1	stripe	thick
Station #2	rotate	135
Station #3	punch	square 1

Prediction:

AR

Right? Y N

Reverse the order of the machines in Assembly line B.
Will the same product be produced?

Assembly line BR	Machine	Selections
Station #1	punch	circle 2
Station #2	rotate	90
Station #3	stripe	medium
Station #4	rotate	180

Prediction:

BR

Right? Y N

EXERCISE 3

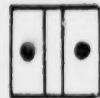
Choose a partner and take turns meeting the computer-generated challenges. The program will give you an unlimited number of tries to reproduce each product offered as a challenge.

BUILD A FACTORY
ANSWERS TO LABORATORY EXERCISES

1A.



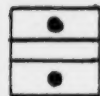
1B.



2AR.



2BR.



BUILD A FACTORY HOMEWORK EXERCISES

1. Predict the product the assembly line will produce. (You will check your predictions in the next class period.)

Assembly line A	Machine	Selections
Station #1	punch	circle 2
Station #2	rotate	90
Station #3	stripe	medium

Prediction:

A

Right? Y N

Prediction:

Assembly line B	Machine	Selections
Station #1	punch	square 3
Station #2	rotate	180
Station #3	stripe	thin

B

Right? Y N

Prediction:

Assembly line C	Machine	Selections
Station #1	rotate	45
Station #2	stripe	thin
Station #3	rotate	90
Station #4	punch	circle 3

C

Right? Y N

Prediction:

Assembly line D	Machine	Selections
Station #1	stripe	thick
Station #2	punch	square 2
Station #3	rotate	135
Station #4	stripe	thin

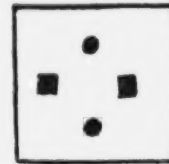
D

Right? Y N

2. Set up an assembly line that will produce the given product.

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		

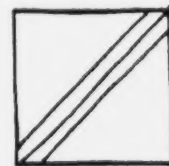
Product A:



Check? Y N

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		

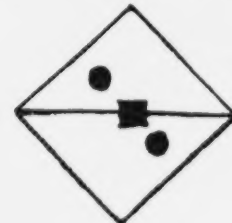
Product B:



Check? Y N

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		

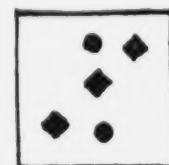
Product C:



Check? Y N

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		

Product D:



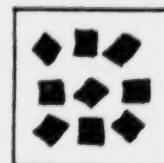
Check? Y N

EXERCISE 3

a) Set up an assembly line to produce the product.

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		
Station #5		
Station #6		
Station #7		
Station #8		

Product a:



Check? Y N

b) CHALLENGE: Produce the product given in Part (a) another way.

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		
Station #5		
Station #6		
Station #7		

Product b:

Check? Y N

BUILD A FACTORY
ANSWERS TO HOMEWORK EXERCISES

1. Predict the product that the assembly line will produce.
(You will check your predictions in the next class period).

Assembly line A	Machine	Selections
Station #1	punch	circle 2
Station #2	rotate	90
Station #3	stripe	medium

Prediction:



A

Right? Y N

Assembly line B	Machine	Selections
Station #1	punch	square 3
Station #2	rotate	180
Station #3	stripe	thin

Prediction:



B

Right? Y N

Assembly line C	Machine	Selections
Station #1	rotate	45
Station #2	stripe	thin
Station #3	rotate	90
Station #4	punch	circle 3

Prediction:

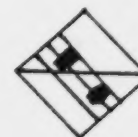


C

Right? Y N

Assembly line D	Selections
machine #1	stripe thick
machine #2	punch square 2
machine #3	rotate 135
machine #4	stripe thin

Prediction:



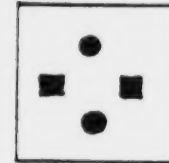
D

Right? Y N

2. Set up an assembly line to produce the product.

Your assembly line	Machine	Selections
Station #1	punch	circle two
Station #2	rotate	90
Station #3	punch	square two

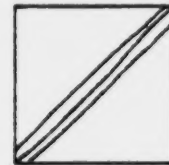
Product A:



Check? Y N

Your assembly line	Machine	Selections
Station #1	rotate	45
Station #2	stripe	thick
Station #3	rotate	45

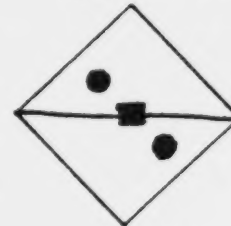
Product B:



Check? Y N

Your assembly line	Machine	Selections
Station #1	punch	circle two
Station #2	rotate	135
Station #3	punch	square one
Station #4	stripe	thin

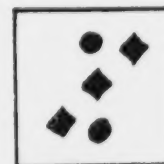
Product C:



Check? Y N

Your assembly line	Machine	Selections
Station #1	punch	circle two
Station #2	rotate	45
Station #3	punch	square three
Station #4	rotate	45

Product D:



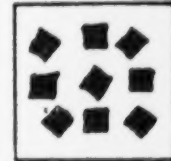
Check? Y N

3.

a) Set up an assembly line to produce the product.

Your assembly line	Machine	Selections
Station #1	punch	square two
Station #2	rotate	45
Station #3	punch	square three
Station #4	rotate	45
Station #5	punch	square two
Station #6	rotate	45
Station #7	punch	square two
Station #8		

Product a:



Check? Y N

b) CHALLENGE: Produce the product given in Part (a) another way.

Your assembly line	Machine	Selections
Station#1	punch	square two
Station #2	rotate	90
Station #3	punch	square two
Station #4	rotate	45
Station #5	punch	square three
Station #6	rotate	90
Station #7	punch	square two
Station #8		

Product b:

Check? Y N

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: MAKE A PRODUCT

OBJECTIVE:

The student will construct a machine assembly line to reproduce a product shown by the program.

COURSES: General Math, Career Math, Applied Math

LEVEL: low

SOFTWARE: THE FACTORY

TIME: two class periods

PREREQUISITE SKILLS/CONCEPTS:

Lesson 1 Punching and Striping

Lesson 2 Rotation

Lesson 3 Build a Factory

FORMAT: laboratory

GENERAL DESCRIPTION OF LAB ACTIVITY:

The program displays a finished product that has been produced on an assembly line containing several machines. Students then design a sequence of machines (with appropriate operations) that will reproduce the given product. The program offers three levels of challenges: easy -- two stations; medium -- up to four stations; hard -- up to eight stations.

MAKE A PRODUCT
LABORATORY EXERCISES
LEVEL: EASY

Making Menu Selections

Under the direction of your teacher, load and boot
The Factory program. The Main Menu will appear.
Select #3: Making a Product.

The next screen will ask: What level challenge do you want?
easy, medium, or hard Select easy. RETURN

The next screen will state:

Your challenge - make this product.
(The picture of a finished product will appear. Then an
8-station assembly line will appear.)

Choose your machine(s) with the necessary options for
reproducing the given finished product.

Your product then will appear on the screen next to
the given product and one of the following will occur:

If your product is wrong, the computer will write:
Your product has a flaw... hit RETURN to continue.
The next screen will ask:
Try making this product again? yes - no.
If you answer yes, the program will give you an
unlimited number of tries to match the given product.

If your product is correct, the program will state:
You made the product correctly! ... press RETURN to
continue.

The next screen will ask:
Do you want to make another product? yes - no
Select yes. RETURN

LAB EXERCISE 1

For this lab exercise, choose easy, and RETURN. A given product will be shown on the screen. You have up to 3 tries to design an assembly line to reproduce this product using up to two stations.

Trial 1.

Your assembly line	Machine	Selections
Station #1		
Station #2		

Right? Y N

Trial 2.

Your assembly line	Machine	Selections
Station #1		
Station #2		

Right? Y N

Trial 3.

Your assembly line	Machine	Selections
Station #1		
Station #2		

Right? Y N

MAKE A PRODUCT
LABORATORY EXERCISES
LEVEL: MEDIUM

LAB EXERCISE 2

From the main menu of The Factory, select #3, Making a Product. For this lab exercise, choose medium. A given product will be shown on the screen. You have up to 3 tries to design an assembly line to reproduce this product using up to four stations. The challenge is to reproduce the product using the fewest stations.

Trial 1.

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		

Right? Y N

Trial 2.

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		

Right? Y N

Trial 3.

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		
Station #5		

Right? Y N

MAKE A PRODUCT
LABORATORY EXERCISES
LEVEL:HARD

LAB EXERCISE 3

From the main menu of The Factory, select #3, Making a Product.
For this lab exercise, choose hard.

You will be given a product on the screen and up to 3 tries
to design an assembly line to reproduce this product.

The challenge is to reproduce the product using no more than
eight stations.

Trial 1.

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		
Station #5		
Station #6		
Station #7		
Station #8		

Right? Y N

Trial 2.

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		
Station #5		
Station #6		
Station #7		
Station #8		

Right? Y N

Trial 3.

Your assembly line	Machine	Selections
Station #1		
Station #2		
Station #3		
Station #4		
Station #5		
Station #6		
Station #7		
Station #8		

Right? Y N

TITLE: GEARs: STRATEGIES IN PROBLEM SOLVING

PUBLISHER: Sunburst Communications, Inc.
39 Washington Ave.
Pleasantville, NY 10570

PRICE: \$ 65.00
10-Disk Lab Pack: \$195.00

COMPUTERS: Apple II, II+, IIc, IIe, IIGS
Commodore 64, IBM PC or PCjr
Tandy 1000, TRS80 Color
(Apple version reviewed)

PIMMS RATING: Very Good

PROGRAM IS APPROPRIATE FOR:

Course: General, Tech, or Applied
Math, 7th, 8th grade math

Ability Level:
Middle School: average
High School: low/average

Format: discovery/laboratory, game, class
demo/instructional

CHARACTERISTICS: disk dependent between menus
disk independent within a menu
user friendly, good graphics
support materials, documentation

ADDITIONAL PERIPHERALS NEEDED: none

REVIEW AND DESCRIPTION OF THE PROGRAM:

GEARS is a software package which allows students to construct gear "trains" composed of a series of gears which drive one another. Students select each gear in the train by specifying the number of teeth and position of the gear. Then, given the number of rotations the first gear will turn, the student must predict the number of times the last gear will rotate.

The program is divided into three sections:

- 1) Building a Gear Train
 - students experiment by building gear trains
 - students challenge other students to duplicate the results of their gear trains
- 2) Solve a Gear Puzzle
 - the computer software program poses a gear puzzle and the student must construct a gear train which matches the computer's results in terms of direction and number of rotations
- 3) Build a Gear Factory
 - this section is similar to "Solve a Gear Puzzle" except that the computer program asks the student to produce products which vary as a result of the direction of the rotation of the final gear in the train

Each section of the program is supported by on-disk instructions. The user input is simple enough that even computer novices will quickly understand its operation. The gear train construction sequence allows students to remove or replace gears before the train is engaged.

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: GEAR ROTATIONS

OBJECTIVES:

The student should be able to:

- determine the direction of rotation of any gear in a gear train given the direction of one gear
- predict the impact on the number of rotations of the final gear when additional gears are added to the middle of a train

COURSES: Applied Math, General Math, Tech Math
7th or 8th grade math

SOFTWARE: GEARS: STRATEGIES IN PROBLEM SOLVING

TIME: one or two class periods

PREREQUISITE SKILLS/CONCEPTS:

The student should

understand the meaning of the terms
rotation, clockwise, counterclockwise,
worm gear, gear train.

FORMAT: discovery/laboratory

GENERAL DESCRIPTION OF THE LAB ACTIVITY

Before beginning the computer lab activity, the teacher should take a moment to familiarize the students with the terms rotation, clockwise, counterclockwise, worm gear, gear train, and teeth on a gear. It is also recommended that the teacher help each student boot the computer with the software prior to student use. If the students are responsible with the computers and the software, this step may not be necessary as directions to boot the program are included in the laboratory exercise sheets. The teacher also may wish to guide the class step by step through the first gear train construction at the start of the lab. Teachers are encouraged to be flexible in allowing students to complete charts on the basis of discovered patterns without building each gear train.

GEAR ROTATIONS LABORATORY EXERCISES

Getting Started

1. Begin by carefully placing the GEARS diskette into the computer's disk drive. Turn the computer on.
2. At the first menu, select 1 - Build a Gear Train. (You can change the selection by either typing the number of your choice or using the right and left arrow keys.) Press the Return Key.
3. Answer YES to the question Would you like instructions? Read the page of instructions, then press the Return Key to proceed.
4. Select the number of teeth. (You are asked to select the number of teeth for the first gear.) Use the right arrow key to select 9. If you overshoot the '9' use the left arrow key to move back. Press the Return Key to finalize your selection.
5. Notice that a gear with 9 teeth is now located next to the vertical "worm gear" at the top of the screen. (Note that only eight of the nine teeth are visible - one is hidden by the device which holds the gear.) We now wish to add a second gear to the train. In response to the question, "What do you want to do next?", select the Add option by using the right and left arrow keys, then press Return.
6. Select '5' for the number of teeth on the second gear. Use the same procedure as for selecting the number of teeth on the first gear. Press the Return Key to proceed.
7. In a moment we will instruct the computer to "Run" the gear train. Before we do, get ready to observe and record the direction in which the two gears are rotating -- clockwise or counterclockwise.
8. When the program asks, "What do you want to do next?" select Run. (Use the arrow Keys as needed and press Return.)
9. As the program runs, notice the direction the worm gear drives the first (9-tooth) gear and the second (5-tooth) gear. Is the 9-tooth gear rotating clockwise (CW) or counterclockwise (CCW)?
 _____ (1) Is the 5-tooth gear rotating CW or CCW? _____ (2)

The box beneath the first gear is keeping track of the number of rotations of this gear. Similarly, the box beneath the second (5-tooth) gear is recording the number of rotations of the second gear. When the run is complete, record the final tally information below.

# of teeth	direction of first gear	# of rotations first gear	# of teeth	direction of second gear	# of rotations second gear
9	_____	_____	5	_____	_____ (3)

10. Answer 'No' (Use the right arrow key.) to the question "Would you like to challenge someone?" then press Return.

11. Answer 'Yes' to the question, "Build another gear train?" then press Return.

12. This time build a gear train with 9 teeth on the first gear and 7 teeth on the second gear. Reread instructions 5 - 9 above if you've forgotten any of the steps.

Record the results of this trial below.

# of teeth	direction of first gear	# of rotations first gear	# of teeth	direction of second gear	# of rotations second gear
9	_____	_____	7	_____	_____ (4)

12. Build three more gear trains as described below. Give the first gear 9 teeth while varying the number of teeth in the second gear from 9 to 11 to 13. Record the results in the spaces provided.

# of teeth	direction of first gear	# of rotations first gear	# of teeth	direction of second gear	# of rotations second gear
9	_____	10	9	_____	_____ (5)
9	_____	10	11	_____	_____ (6)
9	_____	10	13	_____	_____ (7)

(8) In what direction does the first gear rotate?

(9) In what direction does the second gear rotate?

(10) Which second gear completes the most rotations?

(11) What does the number of rotations of the second gear seem to depend upon?

Construct five three-gear trains as described below. A gear train labeled 9-5-5 will have three gears. The first gear will have 9 teeth, the second gear will have 5 teeth and the third gear will have 5 teeth.

While the train is running, pay special attention to the direction each gear is rotating. When the gear train finishes its run the number of rotations will be recorded on the screen, as will the direction of the final gear.

Gear Train	Direction of Rotation			# of Rotations	
	First Gear	Second Gear	Third Gear	First Gear	Third Gear
9-5-5	CCW	_____	_____	10	_____ (12)
9-7-5	CCW	_____	_____	10	_____ (13)
9-9-5	CCW	_____	_____	10	_____ (14)
9-11-5	CCW	_____	_____	10	_____ (15)
9-13-5	CCW	_____	_____	10	_____ (16)

Construct the following four-gear trains. Note the directions of the gears and record the results in the table below.

Gear Train	First Gear	Direction of Rotation			# of Rotations	
		Second Gear	Third Gear	Fourth Gear	First Gear	Fourth Gear
9-5-7-5	CCW	_____	_____	_____	10	_____ (17)
9-7-13-5	CCW	_____	_____	_____	10	_____ (18)
9-11-5-5	CCW	_____	_____	_____	10	_____ (19)
9-13-11-5	CCW	_____	_____	_____	10	_____ (20)

(21) What effect do the middle gears have on the number of rotations of the final gear?

(22) What effect do the middle gears have on the direction of rotation of the final gear?

If the first gear rotates CCW, then

the second gear will rotate _____, (23)

the third gear will rotate _____, (24)

and the fourth gear will rotate _____. (25)

(26) In what direction do you think a fifth gear would rotate?

(27) If the first gear in the train has 9 teeth and the last gear has 5 teeth then the last gear will rotate _____ times for every 10 rotations of the 9-tooth gear.

GEAR ROTATIONS ANSWERS TO LABORATORY EXERCISES

(1) CCW

(2) CW

# of teeth	direction of first gear	# of rotations first gear	# of teeth	direction of second gear	# of rotations second gear
9	CCW	10	5	CW	18 (3)
9	CCW	10	7	CW	12.9 (4)
9	CCW	10	9	CW	10 (5)
9	CCW	10	11	CW	8.2 (6)
9	CCW	10	13	CW	6.9 (7)

(8) CCW

(9) CW

(10) The gear with 5 teeth makes the greatest number of rotations.

(11) The number of rotations of the last gear seems to depend upon the number of teeth on the first gear. (The more teeth on the last gear the fewer rotations.)

Gear Train	Direction of Rotation			# of Rotations	
	First Gear	Second Gear	Third Gear	First Gear	Third Gear
9-5-5	CCW	CW	CCW	10	18 (12)
9-7-5	CCW	CW	CCW	10	18 (13)
9-9-5	CCW	CW	CCW	10	18 (14)
9-11-5	CCW	CW	CCW	10	18 (15)
9-13-5	CCW	CW	CCW	10	18 (16)

Gear Train	First Gear	Direction of Rotation			# of Rotations	
		Second Gear	Third Gear	Fourth Gear	First Gear	Fourth Gear
9-5-7-5	CCW	__CW__	__CCW__	__CW__	10	__18__(17)
9-7-13-5	CCW	__CW__	__CCW__	__CW__	10	__18__(18)
9-11-5-5	CCW	__CW__	__CCW__	__CW__	10	__18__(19)
9-13-11-5	CCW	__CW__	__CCW__	__CW__	10	__18__(20)

(21) The middle gears have no effect on the number of rotations of the final gear.

(22) Each time a middle gear is added, the rotation of the final gears switches from CW to CCW or vice versa.

(23) CW

(24) CCW

(25) CW

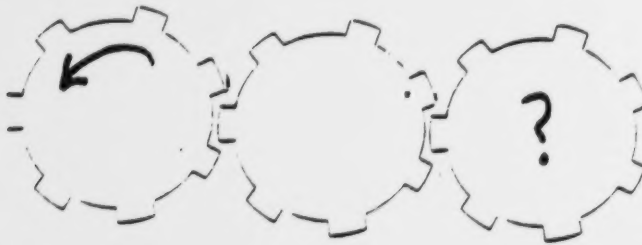
(26) CCW

(27) 18

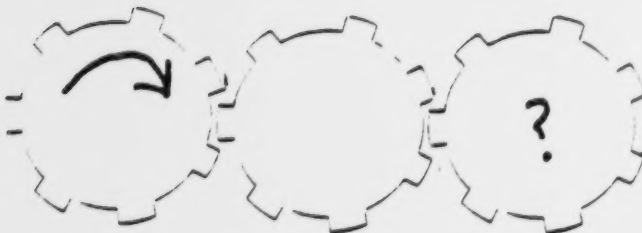
GEAR ROTATIONS
HOMEWORK EXERCISES

Instructions: Given the direction of one of the gears in the gear train, show the direction of rotation for the gear marked with the question mark.

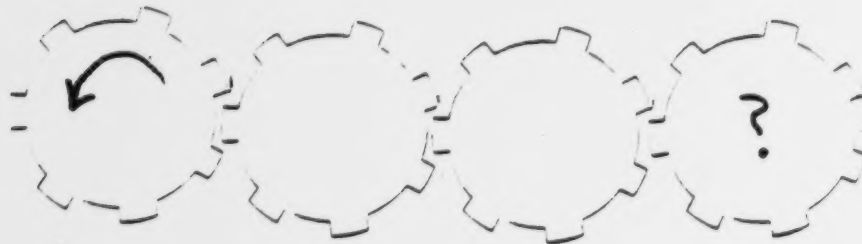
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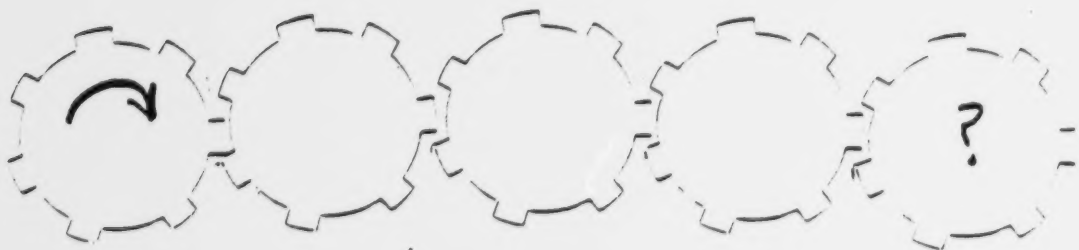
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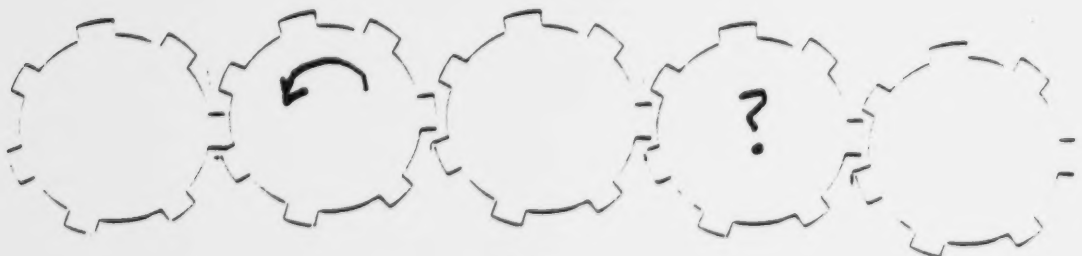
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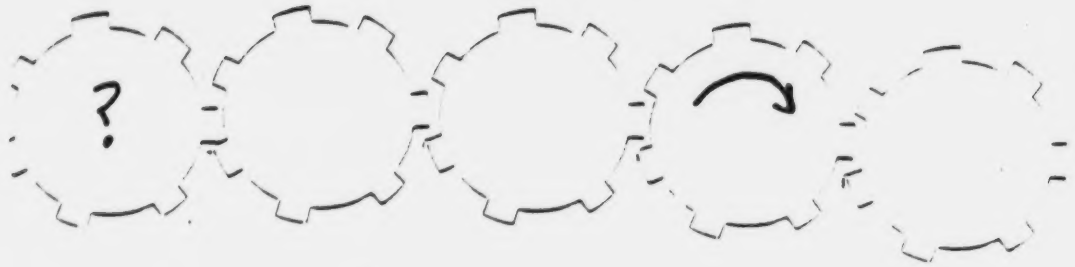
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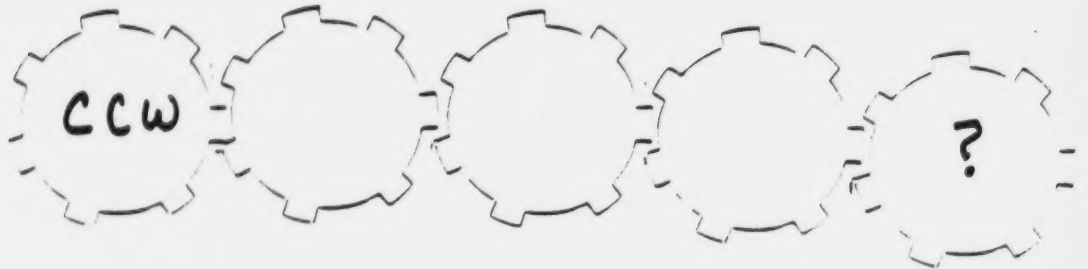
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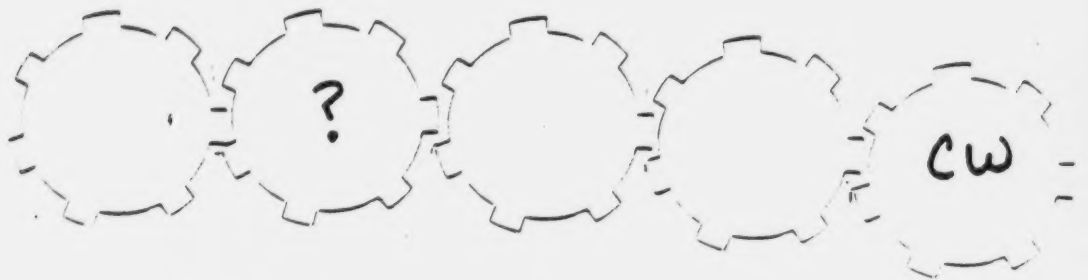
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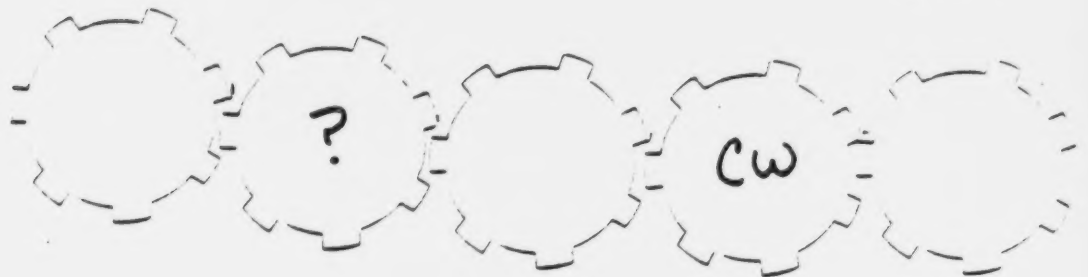
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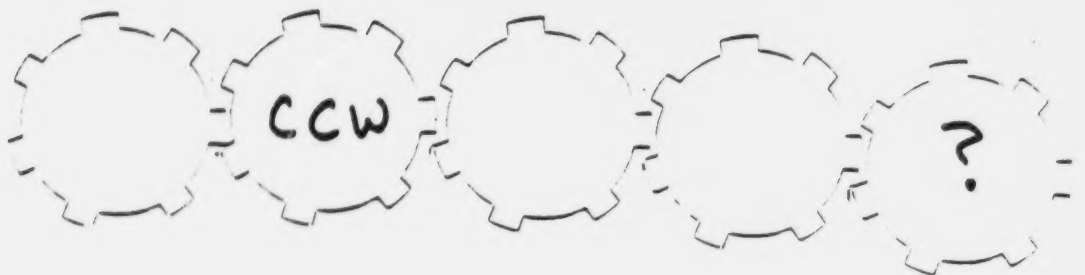
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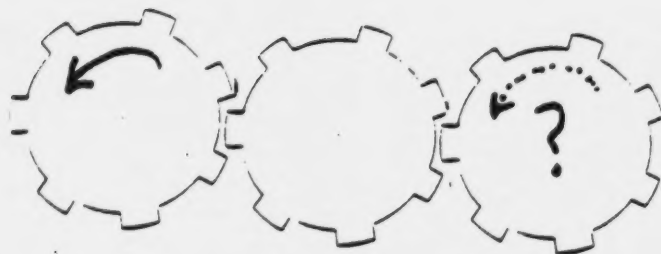


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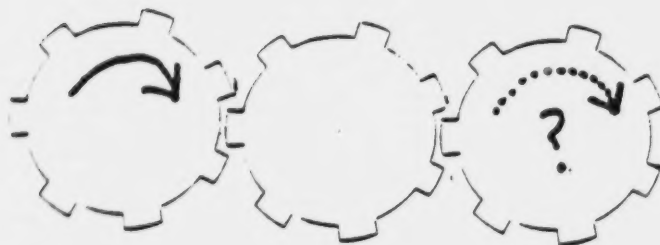


GEAR ROTATIONS
ANSWERS TO HOMEWORK EXERCISES

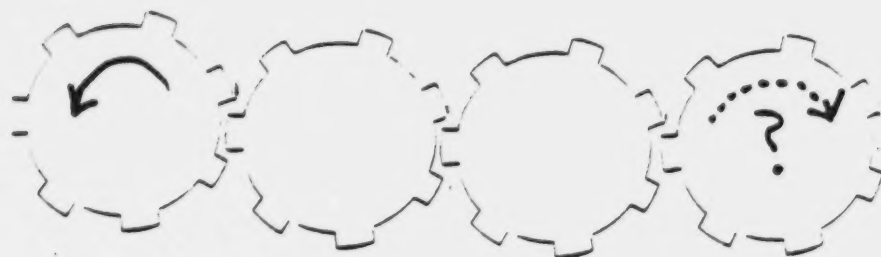
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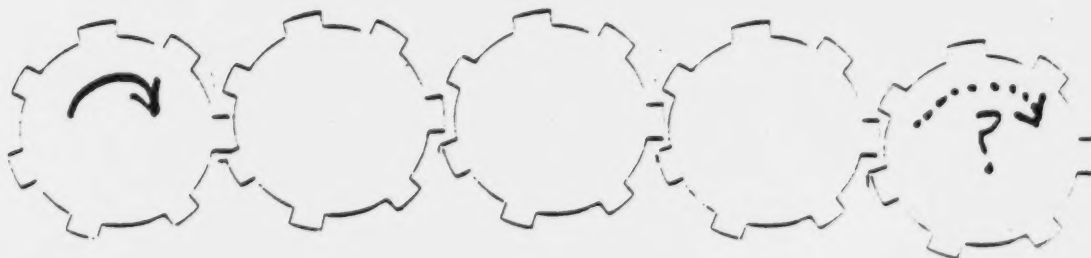
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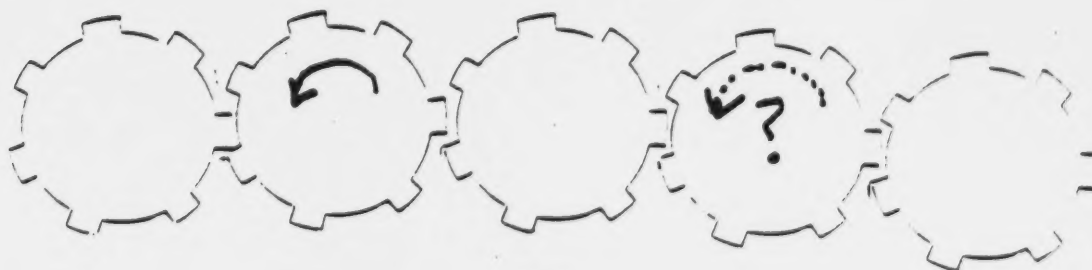
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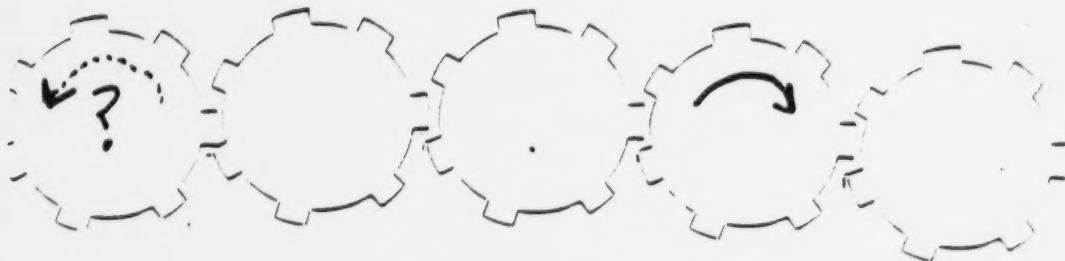


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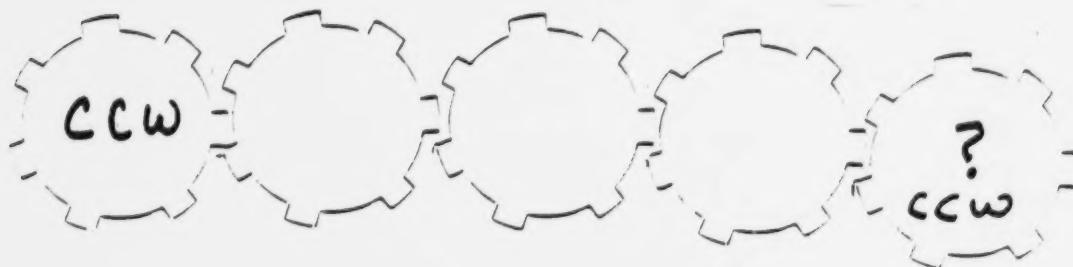


GEAR ROTATIONS ANSWERS TO HOMEWORK EXERCISES

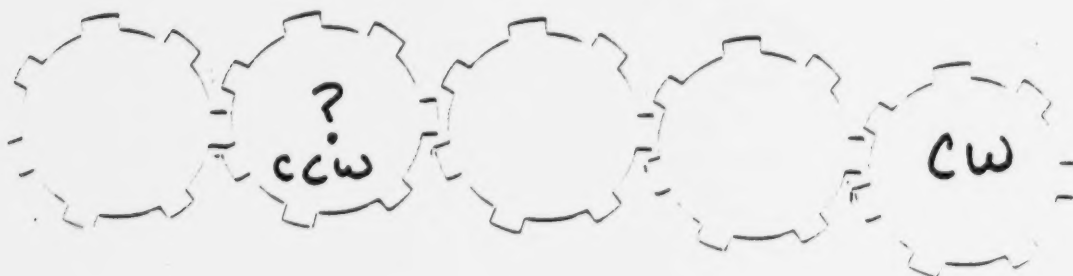
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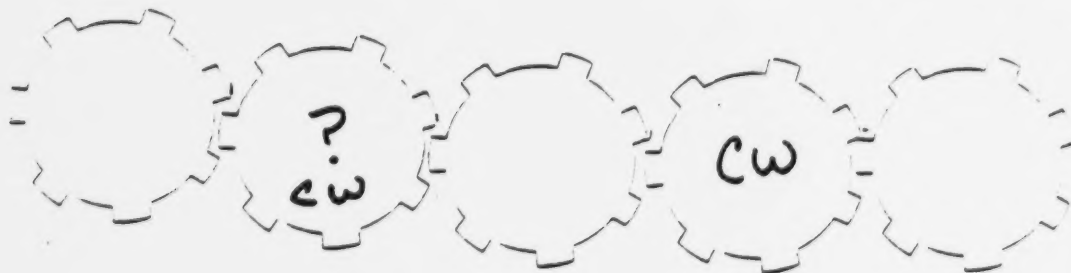
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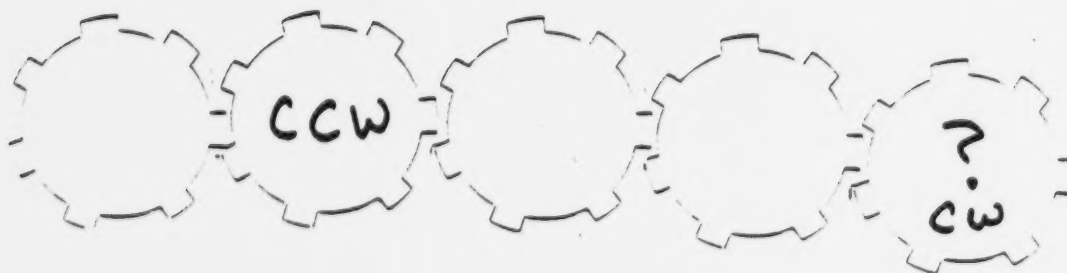
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COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: PREDICTING THE NUMBER OF ROTATIONS

OBJECTIVE:

The student should be able to correctly predict the number of rotations of the final gear in a gear train given the number of teeth in each of the gears and the number of rotations of the first gear.

COURSE: Applied Math, General Math, Tech Math
7th and 8th grade math

SOFTWARE: GEARS: STRATEGIES IN PROBLEM SOLVING

TIME: one class period

PREREQUISITE SKILLS/CONCEPTS:

The student should be able to

- use the software package
- recognize the ratio symbol, :
- divide whole numbers
- round quotients to the nearest hundredth
- multiply whole numbers and decimals

FORMAT: discovery/laboratory

GENERAL DESCRIPTION OF THE LAB ACTIVITY

Progressing from building and analyzing simple two-gear trains to longer trains, the students will develop and use a formula to predict the direction and number of rotations for the last gear in the train.

[The teacher may wish to encourage or discourage the use of electronic calculators during this activity depending upon the teacher's specific objectives for the class. This activity presumes some knowledge from the GEAR ROTATIONS laboratory, so that lesson is a recommended prerequisite.]

PREDICTING THE NUMBER OF ROTATIONS
LABORATORY EXERCISES

Use the GEARS software program. At the first menu select 1 - Build a Gear Train. (Use the right or left arrow keys on the keyboard to indicate your choice.)

Your first task will require constructing five two-gear trains as indicated below. (Remember that 7-5 means to construct the train with a 7-tooth gear and a 5-tooth gear in that order.) Record the number of rotations of the final gear in the space provided in the chart below.

Gear Train	# of rotations of the first gear	# of rotations of the second gear
(1) 7-5	10	_____
(2) 7-7	10	_____
(3) 7-9	10	_____
(4) 7-11	10	_____
(5) 7-13	10	_____

(6) What happens to the number of rotations of the second gear as the number of teeth on the second gear increases?

(7) Discuss the relationship between the number of teeth on the gears which form the train and the number of rotations of the second gear.

Convert the ratio to decimal form rounded to the nearest hundredth.
Write your answers in the spaces provided.

(8) 7:5 _____

(9) 7:7 _____

(10) 7:9 _____

(11) 7:11 _____

(12) 7:13 _____

Compare your responses to questions 1-6 with your responses to questions 8-12.

(13) What is the relationship between these two sets of responses?

Use what you've learned about the relationship between the ratios and the number of rotations of the final gear in the gear train: predict the number of rotations of the final gear in a 9-5 gear train. (Assume that the first gear rotates 10 times.)

Write your prediction here. _____ (14)

Use the computer program to construct a 9-5 gear train. Run this train and record the number of rotations of the final gear here.
_____ (15)

Was your prediction correct?

Use the answer you obtained above. Predict the number of rotations of the final gear in a 9-11-5 gear train.

Write your prediction here. _____ (16)

Now construct a 9-11-5 gear train using the computer program. Note the number of rotations of the final gear. _____ (17)

Was your prediction correct?

(Did you remember that the "middle" gear has no effect on the number of rotations of the final gear?)

Without constructing a gear train, predict the number of rotations of the final gear in a 9-11-5-13 gear train.

Write your prediction here. _____ (18)

Now construct a 9-11-5-13 gear train using the computer program. Write the number of rotations of the final gear here. _____ (19)

Was your prediction correct?

It is important to learn to express our rule for making predictions about the number of rotations of gears in a gear train as a formula. If we can write this rule as a formula, finding solutions for such gear rotation problems will be simplified.

For the sake of writing such a formula, let's assign some variable names.

Use A for the number of teeth in the first gear

Use B for the number of teeth in the last gear

Use X for the number of rotations of the last gear

Remember, the first gear in the GEARS program rotates 10 times.

Using the variables A, B, and X, and the number 10, write a formula to predict the number of rotations of the final gear (X) below. (If you can not figure out how to write this formula, please consult your teacher.)

X =

Compare your formula with the one given on the next page. If it is the same, you're ready to become a Gear Master. If it is different, you should consult your teacher.

The correct formula for predicting the number of rotations of the final gear in a gear train is

$$X = (A / B) \times 10 \quad \text{OR} \quad X = (A \times 10) / B$$

where A is the number of teeth on the first gear

B is the number of teeth on the last gear

X is the number of rotations of the last gear

and 10 is the number of rotations of the first gear

The formula is very important in the gear rotation problems which are yet to come. If you do not understand the formula it is very important that you ask your teacher to explain it to you now!

If you do understand the gear rotation formula, consider this. The formula

$$X = (A / B) \times 10$$

correctly predicts the number of rotations of the final gear in a gear train when the first gear rotates exactly 10 times.

What formula do you think would predict the number of rotations of the final gear if the first gear were to rotate only 8 times?

Write your formula here. $X = \underline{\hspace{2cm}}$ (20)

What formula do you think would predict the number of rotations of the final gear if the first gear were to rotate 15 times?

Write your formula here. $X = \underline{\hspace{2cm}}$ (21)

What formula do you think would predict the number of rotations of the final gear if the first gear were to rotate N times?

Write your formula here. $X = \underline{\hspace{2cm}}$ (22)

Compare the formula you've written above with the one at the top of the next page. If it is not the same consult your teacher.

The correct formula for predicting the number of rotations of the final gear of a gear train is

$$X = (A / B) \times N$$

where A is the number of teeth on the first gear

B is the number of teeth on the last gear

X is the number of rotations of the last gear

and N is the number of rotations of the first gear

This formula also may be written

$$X = (A \times N) / B$$

Given the number of teeth in the first and last gears and the number of rotations of the first gear in a gear train, use either form of the formula to help you predict the number of rotations of the final gear in the train.

(23) Number of teeth in the first gear: 9

Number of teeth in the last gear: 11

Number of rotations of the first gear: 10

Number of rotations of the last gear = _____

(24) Number of teeth in the first gear: 13

Number of teeth in the last gear: 7

Number of rotations of the first gear: 8

Number of rotations of the last gear = _____

(25) Gear Train: 7-13-5

Number of rotations of the first gear: 10

Number of rotations of the last gear = _____

(26) Gear Train: 11-5-7-13

Number of rotations of the first gear: 16

Number of rotations of the last gear = _____

PREDICTING THE NUMBER OF ROTATIONS
ANSWERS TO LABORATORY EXERCISES

(1) 14

(2) 10

(3) 7.8

(4) 6.4

(5) 5.4

(6) The number of rotations of the second gear decreases as the number of teeth on the second gear increases.

(7) Students may notice that the decrease in the number of rotations of the second gear is not constant. Many students will notice little or no relationship at this point.

(8) 1.40

(9) 1.00

(10) 0.78

(11) 0.64

(12) 0.54

(13) The first set (the gear rotations) is ten times as great as the second set (the ratios). (Note that the first gear rotated exactly 10 times.)

(14) Student's predictions will vary.

(15) 18

(16) Student's predictions will vary.

(17) 18

(18) Student's predictions will vary.

(19) 6.9

(20) $X = (A / B) \times 8$

(21) $X = (A / B) \times 15$

(22) $X = (A / B) \times N$

(23) 8.2

(24) 14.9

(25) 14

(26) 13.5

PREDICTING THE NUMBER OF ROTATIONS
HOMEWORK EXERCISES

Given the number of teeth in the first and last gears as well as the number of rotations of the first gear, predict the number of rotations of the final gear. Remember, the correct formula for predicting the number of rotations of the final gear of a gear train is

$$X = (A / B) \times N \quad \text{OR} \quad X = (A \times N) / B$$

where A is the number of teeth on the first gear

 B is the number of teeth on the last gear

 X is the number of rotations of the last gear

and N is the number of rotations of the first gear.

1. Number of teeth in the first gear: 7
 Number of teeth in the last gear: 5
 Number of rotations of the first gear: 10

 Number of rotations of the last gear = _____

2. Number of teeth in the first gear: 13
 Number of teeth in the last gear: 9
 Number of rotations of the first gear: 12

 Number of rotations of the last gear = _____

3. Gear Train: 5-13-11
 Number of rotations of the first gear: 10

 Number of rotations of the last gear = _____

4. Gear Train: 9-13-5-7
 Number of rotations of the first gear: 6

 Number of rotations of the last gear = _____

5. Gear Train: 13-7-5
 Number of rotations of the first gear: 18

 Number of rotations of the last gear = _____

6. Gear Train: 9-7-7-11
 Number of rotations of the first gear: 4

 Number of rotations of the last gear = _____

PREDICTING THE NUMBER OF ROTATIONS
ANSWERS TO HOMEWORK EXERCISES

- (1) 14
- (2) 17.3
- (3) 4.5
- (4) 7.7
- (5) 46.8
- (6) 3.3

COMPUTER SOFTWARE LESSON PLAN

TOPIC: SOLVING A GEAR PUZZLE

OBJECTIVE: Given that the first gear makes 10 counterclockwise rotations, the student should be able to build a gear train in which the last gear rotates a stated number of times in a stated direction.

COURSES: Applied Math, General Math, Tech Math

SOFTWARE: Gears: Strategies in Problem Solving

TIME: one or two class periods

PREREQUISITE SKILLS/CONCEPTS:

The student should be able to

- use the software package
- divide whole numbers
- round numbers to the nearest hundredth
- given the direction of one of the gears, predict the direction of rotation of any gear in the train
- read and understand a two-dimensional table

FORMAT: class discussion and discovery/laboratory

GENERAL DISCUSSION OF THE LAB ACTIVITY

The lesson comes with a Teacher's Guide which discusses a possible approach to the activity. The teacher should adjust this guide in accordance with the capabilities and level of mathematical understanding of the students in the class. Student use of an electronic calculator is recommended.

SOLVING A GEAR PUZZLE
LABORATORY EXERCISES
TEACHER'S GUIDE

Begin by discussing how to build a gear train to produce the desired number of rotations. One approach is given below. The use of overhead transparencies might be helpful.

Each student in the class should construct a 'Gear Ratio Chart' similar to the one provided in the Teacher's Guide. A form for this chart is provided in the laboratory exercises. Students should use this chart to help answer questions in the lesson and the homework exercises.

We have discovered the following relationship between the number of teeth on the first and last gears of a gear train and the number of rotations of the first and last teeth on the gear train.

When A is the number of teeth on the first gear

B is the number of teeth on the last gear

X is the number of rotations of the last gear

and N is the number of rotations of the first gear

then $X = (A \times N) / B$ or $X = (A / B) \times N$.

This formula accurately predicts the number of rotations of the last gear in the train when the number of rotations of the first gear and the number of teeth in the first and last gears are known. (Remember, if the train has any "middle gears," they affect only the direction of the final gear, not the number of rotations.)

Suppose however, we wish to construct a gear train which achieves a stated number of rotations for its final gear. We are allowed to pick the gears we want in the train (5-teeth, 7-teeth, 9-teeth, 11-teeth or 13-teeth) and place them in the train in any order to produce the correct number of rotations for the final gear. How can we do it?

Division of both members of the equation by N (N can't be 0 unless the first gear rotates 0 times) gives

$$X / N = A / B .$$

When X and N are known (for the moment let's assume the first gear rotates ten times), it is a simple matter to find A and B since we know each can have only one of five possible whole number values.

For example, suppose we want to produce a gear train whose final gear is to rotate 22 times. We assume the first gear rotates 10 times.

$$22 / 10 = A / B$$

or, $2.2 = A / B .$

Now we must find two gears (having the available numbers of teeth) whose ratio comes closest to 2.2. The solution may require a good bit of guessing on the part of students, so the following table might prove helpful. (Students will create this table as part of the lab activity.)

		Number of Teeth on Last Gear				
		5	7	9	11	13
Number of Teeth on First Gear	5	1	0.71	0.56	0.45	0.38
	7	1.4	1	0.78	0.64	0.54
	9	1.8	1.29	1	0.82	0.69
	11	2.2	1.57	1.22	1	0.85
	13	2.6	1.86	1.44	1.18	1

The ratio we desire is easily found in the table -- the first gear should have eleven teeth and the last gear should have five teeth.

Should we include any "middle gears"? We know these gears will not affect the number of rotations of the last gear but they will change its direction. If we wish the last gear to rotate 22 times clockwise, then no middle gears are required. If we want the last gear to rotate 22 times counterclockwise, then we insert a middle gear. [Discuss the generalizations of inserting any even number of middle gears or any odd number of middle gears.]

Example: Construct a gear train such that the final gear will rotate 15.7 times counterclockwise.

First, if the final gear is to rotate 15.7 times (and the first gear 10 times) then the gear ratio we seek is $15.7/10$, or 1.57. Using the chart we decide to give the first gear 11 teeth and the last gear 7 teeth. Since the first gear always rotates CCW, the second gear will rotate CW and the third CCW. Therefore a middle gear with any number of teeth may be inserted to complete the chain.

SOLVING A GEAR PUZZLE LABORATORY EXERCISES

Part I: Complete the Gear Ratios Chart below. For each entry divide the number of teeth on the first gear by the number of teeth on the last gear. Round each quotient to the nearest hundredth -- for example, if the last gear has seven teeth and the first gear has five teeth, give the gear ratio (5/7) as 0.71.

		Gear Ratios Chart				
		Number of Teeth on Last Gear				
		5	7	9	11	13
Number of Teeth on First Gear	5	1	0.71			
	7					
	9					
	11					
	13					

Part 11: We will now let the computer software program pose some gear train problems for us, but before we do, let's review one example.

Use the gear ratio chart you just completed and the formula $X/N = A/B$ to find a gear train to match the specifications in the problem below. Remember that the first gear in any train will rotate counterclockwise (CCW) exactly 10 times ($N = 10$) and that

A is the number of teeth on the first gear

B is the number of teeth on the last gear

X is the number of rotations of the last gear

and N is the number of rotations of the first gear.

Example: Build a gear train in which the last gear turns counterclockwise and makes 26 rotations.

Solution: First find the gear ratio, X / N . X is 26 (the number of rotations of the last gear) and N is 10 (the number of rotations of the first gear), so $X / N = 2.6$. Now consult the Gear Ratios Chart to find the ratio 2.6. Note that the first gear must contain 13 teeth and the last gear must contain 5 teeth.

Next, if the final gear is to rotate CCW, the train must have three gears (to give the rotational sequence CCW,CW,CCW).

Conclusion: One answer to the problem is a 13-7-5 gear train. (Note that the 7-tooth gear could be a gear with any number of teeth.)

Making Menu Choices

1. At the first menu, select 2 - Solve a Gear Puzzle. (Remember to use the arrow keys to make your selection, then press Return.)
2. When the program asks "Would you like instructions?" answer 'Yes' then press the Return key.
3. Read the page of instructions and press the Return key.
4. Answer 'Easy' to the question "What type of problems do you want?" Press the Return key to continue.
5. A problem now will be posed for you. Copy the information in the space provided below.

* Build a gear train in which the last gear turns _____ and makes _____ revolutions.

Answer 'Yes' to the question "Will you accept this CHALLENGE?" Press the Return key to continue.

6. Now solve the problem by finding the ratio X / N . (Remember, N is 10 and X is the number you filled in the blank above.) Look for appropriate gear pairs in the Gear Ratio Chart. Then decide whether any 'middle' gears are needed to make the last gear turn in the correct direction.

When you've decided on your solution, enter the appropriate gears by using the arrow keys and the Return key for each selection. If you accidentally put a gear in the train you did not want, select the Remove option when the program asks, "What do you want to do next?" When all of the gears are in place, select Run and watch the gear train go.

If your gear train is correct, write your solution here _____ and press the Return key to continue.

If your gear train is not correct, the program will tell you what sort of error you made. Answer 'Yes' to the question "Would you like to try again?" (and press Return). If you can determine how to correct your gear train, construct the correct one now; if not, call on your teacher for help.

When you have the correct gear train, write it here _____ and press the Return key to continue.

7. Select 'Yes' to the question "Solve another gear problem?" Then press the Return key.
8. Select 'Easy' to answer the question "What type of problem do you want?" Then press the Return key to continue.
9. Copy the computer's next CHALLENGE below. Then press the Return key to accept it.

Build a gear train in which the last gear turns _____ and makes _____ revolutions.

10. When you find the correct gear train to solve the computer-generated problem, write the solution below. (Call your teacher if you need some help.)

11. Ask for two more 'Easy' problems. For each problem, write the CHALLENGE below and then your solution to the problem.

- a) Build a gear train in which the last gear turns _____ and makes _____ revolutions.

My gear train solution is _____.

- b) Build a gear train in which the last gear turns _____ and makes _____ revolutions.

My gear train solution is _____.

SOLVING A GEAR PUZZLE
HOMEWORK EXERCISES

Find a gear train which will meet the given requirements.
Remember: The first gear turns counterclockwise and makes 10 rotations.

Use the gear ratio chart and the formula

$$X / N = A / B$$

where A is the number of teeth on the first gear
 B is the number of teeth on the last gear
 X is the number of rotations of the last gear
and N is the number of rotations of the first gear.

1. Build a gear train in which the last gear makes 14 CW rotations.

solution _____

2. Build a gear train in which the last gear makes 15.7 CCW rotations.

solution _____

3. Build a gear train in which the last gear makes 6.9 CW rotations.

solution _____

4. Build a gear train in which the last gear makes 12.2 CW rotations.

solution _____

5. Build a gear train in which the last gear makes 18.6 CCW rotations.

solution _____

6. Build a gear train in which the last gear makes 4.5 CCW rotations.

solution _____

7. Build a gear train in which the last gear makes 10 CW rotations.

solution _____

SOLVING A GEAR PUZZLE
ANSWERS TO HOMEWORK EXERCISES

Solutions below are not necessarily unique. 'Middle gears' which may contain any number of teeth are indicated by ?.

1. 7-5 (or 7-?-?-5)
2. 11-?-7
3. 9-13 (or 9-?-?-13)
4. 11-9 (or 11-?-?-9)
5. 13-?-7
6. 5-?-11
7. X-X (or X-?-?-X) where X is 5, 7, 9, 11 or 13

COMPUTER SOFTWARE LESSON PLAN

TOPIC: DIFFICULT GEAR PUZZLES

OBJECTIVE: The student should be able to build a gear train in which the last gear rotates a stated number of times in a stated direction given that the first gear rotates between 1 and 30 times counterclockwise.

COURSES: Applied Math, General Math, Tech Math

SOFTWARE: Gears: Strategies in Problem Solving

TIME: one or two class periods

PREREQUISITE SKILLS/CONCEPTS:

The student should be able to

- use the software
- divide whole numbers
- round numbers to the nearest hundredth
- predict the direction of a gear's rotation in a gear train given the direction of one of the gears
- read and understand a two-dimensional table

FORMAT: class discussion and computer laboratory

GENERAL DISCUSSION OF THE LAB ACTIVITY

Gear train design is now expanded to provide for other than 10 rotations of the first gear. The lesson builds upon the previous lessons.

The lesson comes with a Teacher's Guide which offers a possible approach to the activity. The teacher should adjust this guide in accordance with the capabilities and level of mathematical understanding of the students in the class. Student use of an electronic calculator is highly recommended.

DIFFICULT GEAR PUZZLES
TEACHER'S GUIDE

So far we have built a number of gear trains from gears which have had 5, 7, 9, 11, and 13 teeth. Also, we have investigated only gear trains in which the first gear has rotated 10 times. If the first gear is allowed to rotate more or less than ten times, the task of solving gear puzzles becomes much more difficult.

Initiate the lesson by reviewing the process for solving a gear puzzle in which the first gear rotates 10 times. Then, making sure that the students have a Gear Ratios Chart and calculator, suggest varying the number of rotations of the first gear to whole numbers other than 10.

For example, suppose we are challenged to build a gear train in which the last gear turns clockwise and makes 9.2 rotations. If the first gear is restricted to making 10 rotations, the problem can not be solved using gears with 5, 7, 9, 11, and 13 teeth. (This is because the gear ratio $9.2/10 = 0.92$ can not be found in our Gear Ratios Chart.) However, we may adjust the number of rotations of the first gear to help find a solution to the problem.

Using the relation $X / N = A / B$ where

A is the number of teeth on the first gear
B is the number of teeth on the last gear
X is the number of rotations of the last gear
and N is the number of rotations of the first gear,

we will find A and B by experimenting with different values of N.
[Note: The program chooses only integer values for N from 1 thru 30.]

when $N = 1$, $X/N = 9.2$ (Much too high to be in our chart!
We need a larger N.)

when $N = 4$, $X/N = 2.3$ (A number within the range of values
in the chart, but it does not appear.)

when $N = 5$, $X/N = 1.84$

when $N = 6$, $X/N = 1.53$

when $N = 7$, $X/N = 1.31$

when $N = 8$, $X/N = 1.15$

when $N = 9$, $X/N = 1.02$

when $N = 10$, $X/N = 0.92$

when $N = 11$, $X/N = 0.84$

when $N = 12$, $X/N = 0.77$

when $N = 13$, $X/N = 0.71$ (Eureka! This one is in the chart!)

The chart suggests using a first gear with 5 teeth and a last gear with 7 teeth when the first gear is set to rotate 13 times. We need include no 'middle' gears since the second gear will turn clockwise as desired.

Other values of N may give the desired outcome for a different gear combination, so multiple answers are possible. (For example, another solution to the problem above could be a 7-13 gear train when the first gear rotates 17 times.)

Example 2: Build a gear train in which the last gear turns counterclockwise and makes 14.9 rotations.

Solution: Evaluate $14.9/N$ for whole number values of N from 1 to 30.

when $N = 1$, $14.9/1 = 14.9$ (Much too high)

when $N = 6$, $14.9/6 = 2.48$

when $N = 7$, $14.9/7 = 2.13$

when $N = 8$, $14.9/8 = 1.86$ (That's the one!)

Conclusion: The train should begin with a 13-tooth gear, which should rotate 8 times, and end with a 7-tooth gear. There should be one 'middle' gear to make the last gear rotate counterclockwise.

DIFFICULT GEAR PUZZLES
LABORATORY EXERCISES

Use your Gear Ratios Chart and a calculator to help solve the gear puzzle. Remember that our gear ratio formula is

$$X / N = A / B$$

where A is the number of teeth on the first gear

 B is the number of teeth on the last gear

 X is the number of rotations of the last gear

and N is the number of rotations of the first gear.

The first gear in any train will rotate counterclockwise (CCW) and its number of rotations may be set at any whole number from 1 through 30.

Example: Build a gear train in which the last gear turns counterclockwise and makes 39 rotations.

Solution: Evaluate X/N where $X = 39$ and N takes on whole number values from 1 through 30.

When $N = 1$, $X/N = 39$ (This number is much too high to be found in our chart. Let's try a larger N .)

When $N = 10$, $X/N = 3.9$ (Still too large.)

When $N = 14$, $X/N = 2.79$

When $N = 15$, $X/N = 2.6$ (2.6 is in the chart! We've found it!)

The gear ratio chart indicates the first gear should have 13 teeth and the last gear should have 5 teeth. (Remember, the first gear must make 15 rotations.)

Next, if the final gear is to rotate CCW, the train must have three gears.

Conclusion: One possible solution to the gear puzzle is the gear train 13-11-5 when the 13-tooth gear rotates 15 times. (Note that the 11-tooth gear could be a gear with any number of teeth.)

LAB EXERCISE

Use the GEARS software package.

1. At the first menu, select 2 - Solve a Gear Puzzle. (Remember to use the arrow keys to make your selection, then press the Return Key.)
2. When the program asks "Would you like instructions?" answer 'No' and then press the Return Key.
3. Answer 'Hard' to the question "What type of problems do you want?" Press the Return Key to continue.
4. Now a problem will be posed for you. Copy the information in the space provided below.

Build a gear train in which the last gear turns _____
and makes _____ revolutions.

Answer 'Yes' to the question "Will you accept this CHALLENGE?" Press the Return Key to continue.

5. First find the ratios X/N . (Remember X is the number you wrote in the blank above and N can be any whole number from 1 through 30.) Look for the appropriate gears in the Gear Ratios Chart. When you've selected the first and last gears and calculated the correct number of rotations for the first gear, decide if any 'middle' gears are needed to make the last gear turn in the correct direction.

It's now time to enter our solution to the puzzle given by the computer program. Use the arrow keys and the Return Key to select the gears you want in the train. If you accidentally put an unwanted gear in the train, select the Remove option when the program asks, "What do you want to do next?" When all of the gears are in place, select the Run option and press the Return Key. The program now requests the number of rotations you would like to assign to the first gear in the train. Enter the value of N you selected from the X/N ratios found in the chart. You can increase and decrease the number by using the arrow keys. (The right arrow key increases the number of rotations, the left key decreases the number of rotations.) When the number of rotations is correct, press the Return Key and watch the gear train go. (Be patient! The program may take a while to run as the value of N increases.)

If your train is correct, write your solution here.

Gear Train _____ The first gear rotates _____ times.

Press the Return Key to continue.

If your gear train is not correct, the program will tell you what sort of error you made. Answer 'Yes' to the question "Would you like to try again?". (Press Return to continue.) If you can determine how to correct your gear train, construct the correct one now. If not, call on your teacher for help.

When you have the correct gear train, write it here.

Gear Train _____ The first gear rotates _____ times.

Press the Return Key to continue.

6. Select 'Yes' to the question "Solve another gear problem?" Then press the Return Key.
7. Select 'Hard' to answer the question "What type of problem do you want?" Then press the Return Key to continue.
9. Copy the computer's next CHALLENGE below. Then press the Return Key to accept it.

Build a gear train in which the last gear turns _____ and makes _____ revolutions.

10. When you find the correct gear train to solve the computer's problem, write the solution below. (Call the teacher if you need some help.)

Gear Train _____ The first gear rotates _____ times.

11. Ask for two more 'Hard' problems. For each problem, write the CHALLENGE below and then your solution to the problem.

- a) Build a gear train in which the last gear turns _____ and makes _____ rotations.

Gear Train _____ The first gear rotates _____ times.

- b) Build a gear train in which the last gear turns _____ and makes _____ rotations.

Gear Train _____ The first gear rotates _____ times.

DIFFICULT GEAR PUZZLES
HOMEWORK EXERCISES

Find a gear train which will meet the given requirements.
Remember: The first gear always turns counterclockwise.

Use the gear ratio chart and the formula $X / N = A / B$

where A is the number of teeth on the first gear
 B is the number of teeth on the last gear
 X is the number of rotations of the last gear
and N is the number of rotations of the first gear.

Remember, you control the number of times the first gear rotates -- any whole number from 1 through 30.

Build a gear train in which the last gear

1. turns clockwise and makes 8.7 rotations.

gear train solution _____ The first gear rotates _____ times.

2. turns counterclockwise and makes 15.4 rotations.

gear train solution _____ The first gear rotates _____ times.

3. turns clockwise and makes 7.6 rotations.

gear train solution _____ The first gear rotates _____ times.

4. turns clockwise and makes 16.8 rotations.

gear train solution _____ The first gear rotates _____ times.

5. turns counterclockwise and makes 3.3 rotations.

gear train solution _____ The first gear rotates _____ times.

DIFFICULT GEAR PUZZLES
ANSWERS TO HOMEWORK EXERCISES

Solutions are not necessarily unique. 'Middle gears' which may contain any number of teeth are indicated by ?.

1. 13-9 (or 13-?-?-9), the first gear makes 6 rotations
2. 7-?-5, the first gear makes 11 rotations
3. 11-13 (or 11-?-?-13), the first gear makes 9 rotations
4. 7-5 (or 7-?-?-5), the first gear makes 12 rotations
5. 9-?-11, the first gear makes 4 rotations

COMPUTER SOFTWARE PROGRAM REVIEW AND DESCRIPTION

TITLE: BUILDING PERSPECTIVE
PUBLISHER: Sunburst Communications
ADDRESS: 39 Washington Ave.
Pleasantville, NY 10570
(800) 431-1934
PRICE: \$65
10-Disk Lab Pack \$195
COMPUTER: Apple II family with at least 64K memory
PIMMS RATING: Very Good
PROGRAM IS APPROPRIATE FOR:

Courses: Middle school, Junior High,
General Math 9-12, Geometry

Ability level: all

Format: classroom demonstration/lab

CHARACTERISTICS: disk-independent

ADDITIONAL PERIPHERALS NEEDED :
color monitor highly desirable but not essential

REVIEW AND DESCRIPTION OF THE PROGRAM:

The program helps students develop visual thinking skills, including spatial perception and spatial reasoning.

The Instructions section may be used to demonstrate the use of the program and its underlying concept. The section provides graphical descriptions in the form of four different ground-level views of a 3x3 array of buildings. The object is to predict the top view (height) of each building.

Further Main Menu choices are: Change Array Size, Play the Game, End.

The smallest array consists of nine buildings arranged in 3 rows by 3 columns (3 X 3). In the 3x3 array only the building in the center is hidden when the array is viewed from all sides at ground level. The 4 X 4 array is much more challenging since as many as four buildings may be hidden from view. (A program option permits removal of one building at a time to gain a better view.) The largest array consists of 5 rows and 5 columns and is probably best not attempted by the class until the students feel comfortable with 4 X 4 arrays and have discussed general procedural strategies.

Overall, students will gain practice with making observations in two dimensions (i.e., of the views given by the program), analyzing them, and relating them in a three-dimensional context.

Where a color monitor is not available, use of the program can serve to help develop visual discrimination skills since the use of shading (rather than color) requires recognition of very subtle differences in textural patterns.

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: PERSPECTIVE WITH A PHYSICAL MODEL

OBJECTIVES:

- given a Cuisenaire rod assortment, construct a physical model from the models shown in the program
- given a physical model, view only the four sides then predict the top view

COURSES: Grades 7,8
General Math 9 - 12

SOFTWARE: Building Perspective

OTHER MATERIALS:

- Cuisenaire Rod Assortment
- 5" X 5" piece of 1-inch styrofoam having holes (5 rows by 5 columns) that measure 1 cm by 1 cm by 1 cm deep
- building clay (may be used as the base instead of the styrofoam)

TIME: up to one class period

PREREQUISITE SKILLS/CONCEPTS :

- identify the pieces in a collection of Cuisenaire rods

FORMAT: discovery lab

GENERAL DESCRIPTION OF THE LAB ACTIVITY:

The general concept of determining a top view from information gained from the four side views of a square array of "buildings" is introduced via construction of a physical model.

Students - working in pairs - construct a physical model of a 3x3 array of buildings and learn to use information from the four Side Views to predict the building heights. Said information is recorded on a "Top View" of the array.

Then students proceed to use the computer program -- first to become familiar with the program, and then to Play the Game using 3x3 arrays. Along the way, students will develop techniques for reading the displays of the views and for recording data. (The next lesson will build upon the informal data-recording techniques permitted in this lesson.)

The teacher may elect to address 3x3 arrays via both the physical and computer models before addressing 4x4 arrays in a corresponding manner.

PERSPECTIVE WITH A PHYSICAL MODEL
LABORATORY ACTIVITY
TEACHER'S GUIDE

PART I

Acquaint the students with the Cuisenaire rods. An overhead projector can prove useful for this purpose.

All Cuisenaire rod assortments come in the following configurations.

Length	Color
1 cm	Natural (will not be used)*
2 cm	Red
3 cm	Light Green
4 cm	Purple
5 cm	Yellow
6 cm	Dark Green
7 cm	Black
8 cm	Brown
9 cm	Blue
10 cm	Orange

* Since all rods used will be pressed one centimeter into either styrofoam or building clay, the 1-cm (natural-colored) rod would not be visible above the surface.

PART II LABORATORY ACTIVITY Teacher/Student Dialogue (15-20 minutes)

1. Teacher: Use your Cuisenaire Rod Assortment. Show us the 2-cm rod.(red)
2. Teacher (holding up the red rod): Put this rod into one of the holes in the styrofoam base (or building clay). You should now have about half of the rod exposed outside the base. Give the height of the rod above the base. (about 1 cm)
3. Since all of the rods will extend into the base about the same distance, how will this affect the lengths of all of these rods in terms of what we actually see? (all will be 1 cm shorter)
4. Teacher: Will one person from each pair please hold aloft the styrofoam base so that both of you can read the words printed on the sides. Turn the base until the word FRONT is facing you. Now put the Light Green rod in the hole farthest to your right. Next, put the Red rod immediately behind the Light Green rod. Now one partner should hold the base aloft so that the other partner sees only the side labelled FRONT. What color rod can you clearly see? (Light Green)
5. Teacher: Now turn the base so that the side labelled RIGHT is facing you. What rods can you clearly see now? (both Light Green and Red)
6. Teacher: We all agree that we gain more information by looking at the base from a second side.
7. Teacher: We are now going to have some fun while we continue to work in pairs. Our first objective is to figure out what the top view will look like after looking only at the four sides. Once you think you have enough information, you may make a guess as to the top view.
8. Teacher: Now I want one partner to place different rods in the base to form a 3 X 3 array of rods while the other partner looks away.
9. Teacher: Once the array is ready, the "other partner" starts by looking at the FRONT view. When finished, that partner turns away again while the first partner turns the base to the RIGHT view. This process is continued for the LEFT view and the BACK view.
10. Teacher: Once this procedure has been completed and a prediction made, change roles and repeat the game.

Part III COMPUTER ACTIVITY (approximately 20 minutes)

The Building Perspective program should be loaded prior to sending students to the computers. A lab pack is most convenient for this purpose.

Students select Instructions from the Main Menu and practice informally to become familiar with making program choices. They then may proceed to Play the Game for 3x3 arrays. When both partners are comfortable with 3 X 3 arrays, they may proceed to 4 x 4 arrays.

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COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: REPRESENTING VISUAL PERSPECTIVE VIA
PHYSICAL, COMPUTER, AND DIAGRAMMATIC MODELS

OBJECTIVE:

Given various two-dimensional representations of a three-dimensional model, construct the three-dimensional model.

COURSES: Grades 7,8; General Math 9 - 12; Geometry

SOFTWARE: Building Perspective

OTHER MATERIALS :

- Cuisenaire Rod Assortment
- 5" X 5" piece of 1-inch styrofoam having holes (5 rows by 5 columns) that measure 1 cm by 1 cm by 1 cm, or building clay (to use as the base instead of the styrofoam), or Multilinks cubes, or Unifix cubes
- worksheet with 3 X 3 Arrays
- worksheet with 4 X 4 Arrays
- worksheet with 5 X 5 Arrays
- graph paper (4x4 per inch)

TIME: up to 1 class period

PREREQUISITE SKILLS/CONCEPTS:

The student should be able to

- identify the pieces in a set of Cuisenaire rods
- use the software program

FORMAT: discussion/laboratory

GENERAL DESCRIPTION OF THE LAB ACTIVITY:

Students analyze their record-keeping techniques from the last lesson and learn to record view information on a series of $n \times n$ array diagrams to help them make a prediction for the top view (i.e., heights). Using these techniques, many students will progress rapidly from 3x3 to 4x4 arrays. The correlation between the physical model and the computer model may be reviewed and extended by constructing a 4x4 array via both techniques.

The playing of the game now may be extended via use of the paper diagrams to give the views, thereby enabling students to design challenge problems for each other "anyplace and anytime".

REPRESENTING VISUAL PERSPECTIVE VIA ... MODELS
LABORATORY ACTIVITY
TEACHER'S GUIDE

Teacher-Directed Discussion (15 - 20 minutes)

1. Teacher: In our previous class period we worked with both physical models (rods or cubes) and computer models. Those models represented buildings and we viewed the models from four different perspectives.
2. Teacher: Recall either of the models (rods or computer): one problem that we had was trying to remember all the information we gained from each view (Front, Right, etc.).
3. Teacher: Tell us how you went about keeping track of all this information. [Encourage various responses. Have students display their methods via the chalkboard or overhead projector.]
4. Teacher: What is it that all methods for remembering seem to have in common? (must keep a written record of some sort)
5. Teacher: What all of you found was that to be successful you had to develop some way to RECORD and to ORGANIZE the information.
6. Teacher: (Very possibly this method already will have been proposed by a student.) Let's now use a blank 3 X 3 Array Worksheet to help keep track of our guesses. (Use a projector or chalkboard.) First, let's label the blank array with FRONT, RIGHT, BACK, LEFT.
7. Teacher: Now let's try to agree on how to enter information about the FRONT view on our blank array. [Generate discussion.]
8. Teacher: It seems that we can record our information in two ways. What are they? (either by color or the height of the building)
9. Teacher: To keep things clear for everyone I'll record the information both ways. [Continue through the rest of views.]
10. Teacher: When we have completed recording our information, what is it that we really have? (top view)
11. Teacher: [Take questions.]

12. Teacher: We also can model this information by using the Cuisenaire rods and the styrofoam base. The problem here is that the colors of the rods are not the same as the colors of the buildings in the computer display. How can we manage this little problem?
(Use the numbers that represent the heights of the buildings.)

[Note: At this point display the projectual that gives the lengths and colors of the Cuisenaire rods. Reiterate that the actual rod length is 1 cm more than the height of the building because the rod will extend into the base approximately 1 cm.]

13. Teacher: Now let's combine the use of the computer and the rods to help predict the top view of a 4 X 4 array. [At this point have the program generate a 4 X 4 array on the computer monitor.]

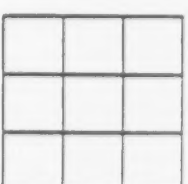
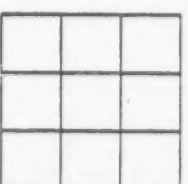
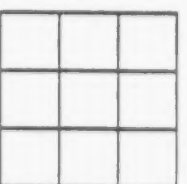
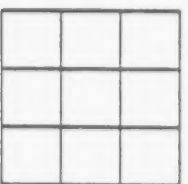
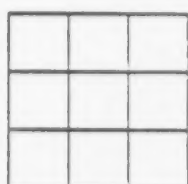
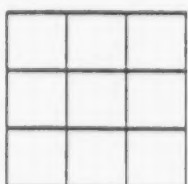
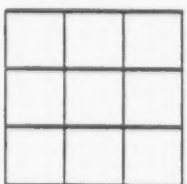
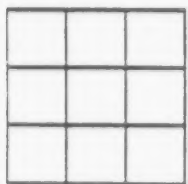
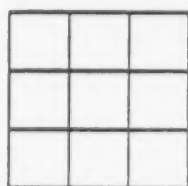
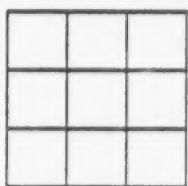
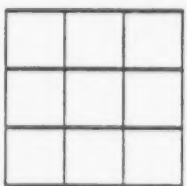
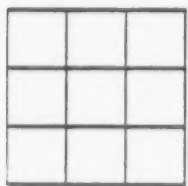
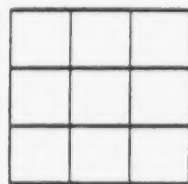
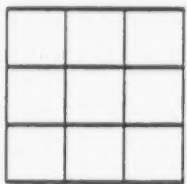
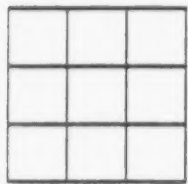
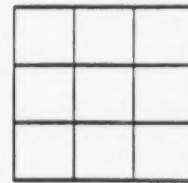
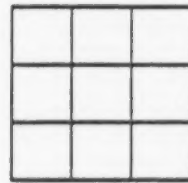
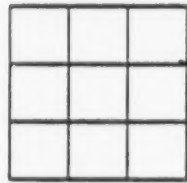
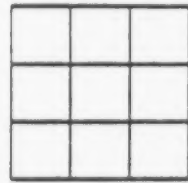
Look at the monitor and tell me what length rod you want me to place in the styrofoam base.

[Go through all four views but, for at least one view, demonstrate the option of removing a building.]

[Establish student pairs or small groups for the Laboratory Exercises. The Building Perspective program should be loaded and the necessary Worksheets issued prior to sending students to the computers.]

REPRESENTING VISUAL PERSPECTIVE VIA ... MODELS
LABORATORY EXERCISES

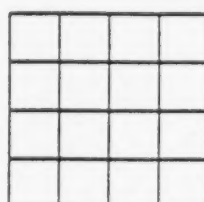
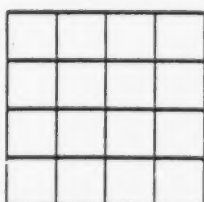
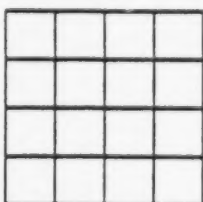
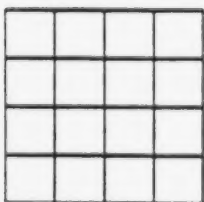
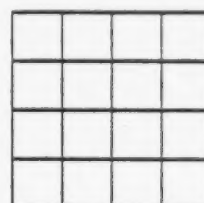
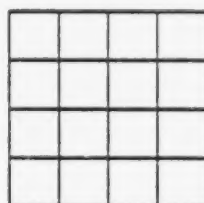
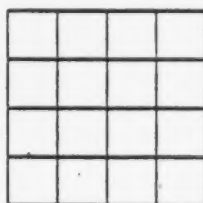
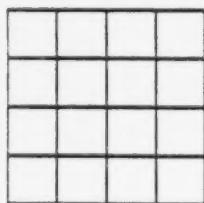
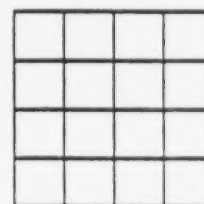
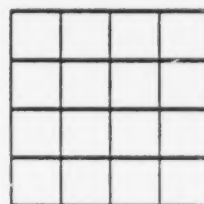
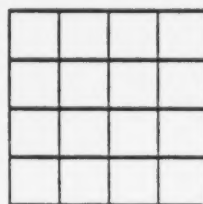
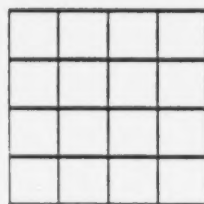
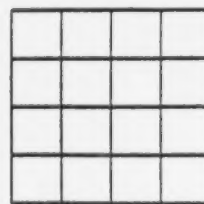
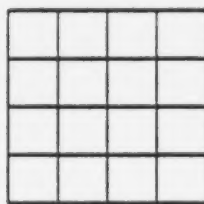
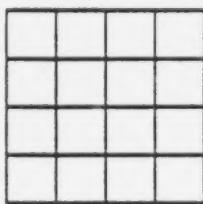
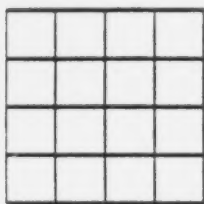
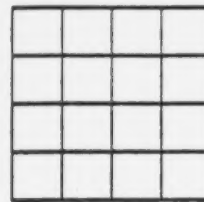
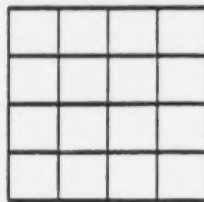
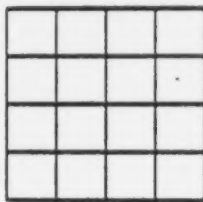
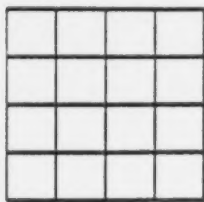
WORKSHEET #1 - 3x3 ARRAYS



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REPRESENTING VISUAL PERSPECTIVE VIA ... MODELS
LABORATORY EXERCISES

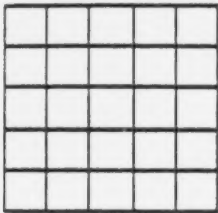
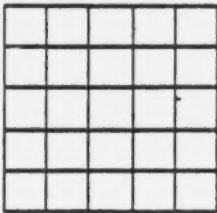
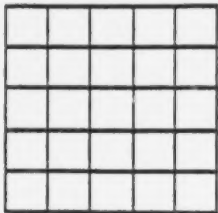
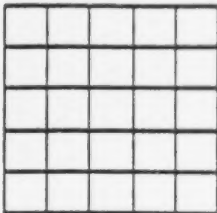
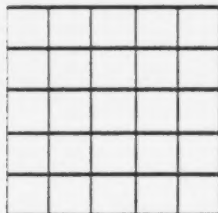
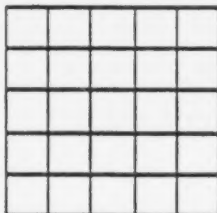
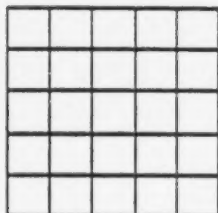
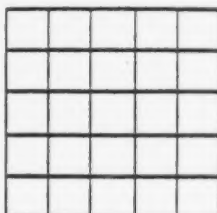
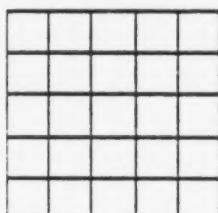
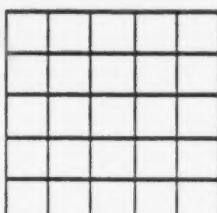
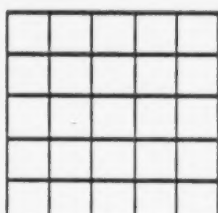
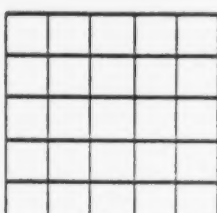
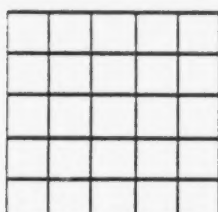
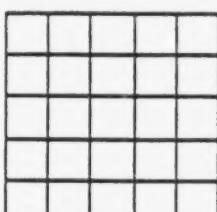
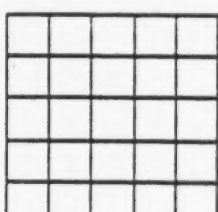
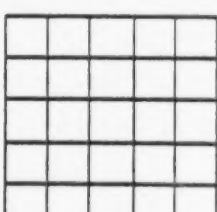
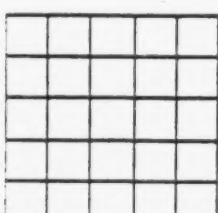
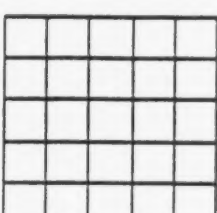
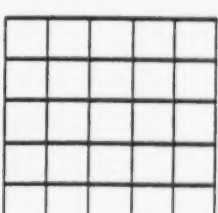
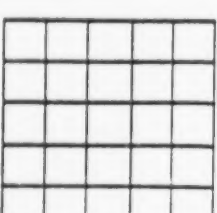
WORKSHEET #2 - 4x4 ARRAYS



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REPRESENTING VISUAL PERSPECTIVE VIA ... MODELS
LABORATORY EXERCISES

WORKSHEET #3 - 5x5 ARRAYS

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REPRESENTING VISUAL PERSPECTIVE VIA . . MODELS
LABORATORY EXERCISES

LAB EXERCISE 1

Work with 3x3 arrays. Use the 3x3 Arrays Worksheet to record information. Do at least as many problems as there are students in the group. When the entire group is ready, proceed to the next exercise.

LAB EXERCISE 2

Work with 4 X 4 arrays. Use either a 4 X 4 Arrays Worksheet to record your information or a physical model (rods with base). The only requirement is that ONE PROBLEM FROM EACH SET MUST BE DONE BOTH WAYS.

LAB EXERCISE 3 (OPTIONAL)

Work with 5x5 arrays. Use either a 5x5 Arrays Worksheet or a physical model to record information. Do at least one problem both ways.

REPRESENTING VISUAL PERSPECTIVE VIA ... MODELS
HOMEWORK EXERCISES

1. Use your data sheets and models from Laboratory Exercise 2 wherein you recorded your information in different ways -- on 4 X 4 array grid, and via a physical model (base with rods, or the cubes). Give written answers to the following questions.

a) Is one method quicker than the other method? Why?

b) Which method do you find to be the most reliable? Why?

c) Do you have a different method that you prefer to use?
Describe it and tell why you like it.

2. Create your own challenge problem: Use a piece of graph paper to show the four views of an array (FRONT, RIGHT, BACK, LEFT) so that they resemble views given on the computer screen by the computer program. You may choose your array size (3 X 3 or 4 X 4). Check your problem to be sure it is a workable one.

REPRESENTING VISUAL PERSPECTIVE VIA ... MODELS
ANSWERS TO HOMEWORK EXERCISES

1. (Student responses will vary. Highlight the merits of useful ideas.)
2. The following questions should prove useful both as a guide to the discussion of techniques in problem creation and to set the stage for the next lesson.

How many students decided to use a 3x3 array? a 4x4? a5x5?

What was your biggest difficulty in creating the four views?

- clearly representing all buildings behind the facing row
- forgetting to represent the buildings behind the facing row
- matching up the different views of the same building

Did you use different colored markers to denote the buildings?

How many different colors do you need?

Did anyone represent the four views without using different colors?

How might we represent the four views without using color?

[Recall the use of physical models. Suggest the use of models drawn on graph paper (next lesson).]

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: REPRESENTING VISUAL PERSPECTIVE VIA GRAPH PAPER MODELS

OBJECTIVES:

To use graph paper with black and white shading techniques to help draw the four face views of a square array of buildings and give detailed building height and placement information.

To use these drawings to create a representation for the top view of the array.

COURSES: Grades 7,8; General Math 9-12; Geometry

SOFTWARE: None required. The lesson is related to the software package Building Perspective.

OTHER MATERIALS:

- graph paper (4x4 per inch)
- Cuisenaire Rod Assortment
- 5" X 5" piece of 1-inch styrofoam having holes (5 rows by 5 columns) that measure 1 cm by 1 cm by 1 cm, or building clay (to be used as the base instead of the styrofoam), or Multilinks cubes, or Unifix cubes
- a 5 X 5 Array Worksheet

TIME: up to 1 class period

PREREQUISITE SKILLS/CONCEPTS:

The student should be familiar with the techniques in the previous lessons.

FORMAT: discussion/laboratory

GENERAL DESCRIPTION OF THE LAB ACTIVITY

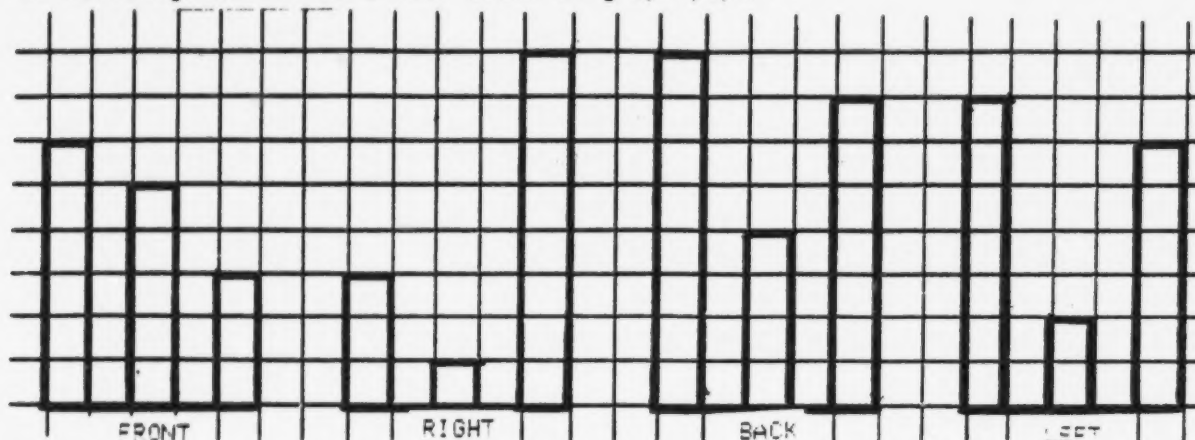
The analysis of a series of four side views of a 3-dimensional structure is carried to a higher level of abstraction by representing the views diagrammatically on graph paper.

Also, the side views-to-top view process is reversed to enable making the side views from a given top view.

REPRESENTING VISUAL PERSPECTIVE WITH GRAPH PAPER LABORATORY EXERCISES

EXAMPLE

Shown below is the "facing row" from each of the four views of a 3x3 array of buildings as it would look drawn on graph paper.



Not everything that can be seen from a view is included in the drawing: Since we are not using color in our drawings, we need a system (code) to indicate what is in the "facing row" and what is behind the "facing row".

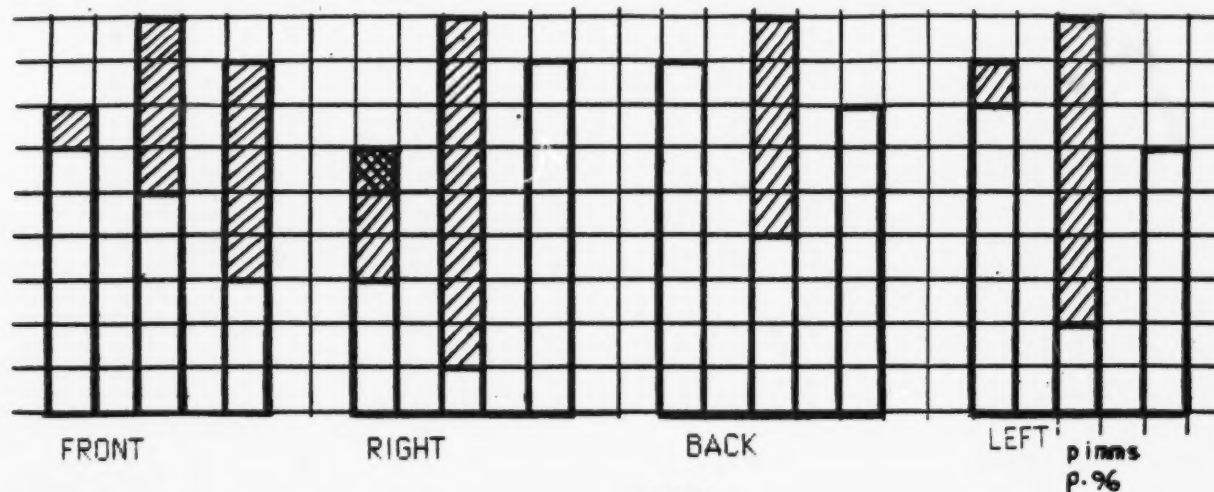
CODE Buildings in the facing row from each view will be represented by a PLAIN BLOCK -- i.e., no lines or shading.

Buildings behind the facing row will be represented by BLOCKS CONTAINING DIAGONAL LINES.

Additional buildings behind the facing row will be represented by BLOCKS CONTAINING CROSS-HATCHING.

Remember, the views and their representations are clearly two-dimensional. A diagonal or cross-hatched design indicates only a building in view, not its row position. That is, a single view does not indicate which building is immediately behind, or two buildings behind, the facing row.

The following diagrams give a fuller picture of the view from each of the four sides of our 3x3 array.



Using these diagrams, we examine the columns (buildings) from left to right and determine the following information.

From the FRONT VIEW

The first column shows a 6-story building in the facing row and a 7-story building in one of the rows behind it.

The second column shows a 5-story building in the facing row and a 9-story building in one of the next two rows behind it.

From the RIGHT VIEW

The first column shows a 3-story building in the facing row with 5- and 6- story buildings behind it. (We know the order of these two buildings because the last building must be the tallest.)

The second column shows a 1-story building in the facing row and a 9-story building in one of the rows behind it.

The third column shows an 8-story building in the facing row and nothing else visible behind it. This means that all the buildings behind the facing row are at most 8 stories tall.

Now that you have the idea, continue with this example by filling in the blanks below.

From the BACK VIEW

The first column shows an 8-story building in the facing row and

_____ .

The second column shows _____

_____ .

The third column shows _____

_____ .

From the LEFT VIEW

The first column _____

_____ .

The second column _____

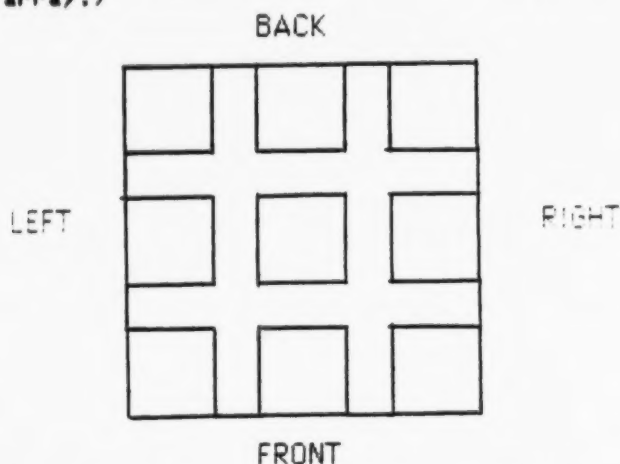
_____ .

The third column _____

_____ .

From the completed four face views we can determine further particular information. Now we are ready to put all the information together to give our TOP VIEW!

To do this, make a diagram. (Each 2x2 square in the diagram represents a building in the 3x3 array.)



Next, put the information from each view together with the information from all the other views, then predict the number of stories in the middle building. Write the predictions in the appropriate 2x2 squares on the diagram.

Write down your predicted heights now for our example-in-progress. Then check your results with your teacher.

There should be sufficient time left in the class period to complete the following Laboratory Exercises.

LAB EXERCISES

1. Each student must create the four views for a 3x3 array and offer it as a challenge to a classmate.

[To do this, use the techniques developed in the Example. Create your own Challenge Problem for a 3x3 array. Start by drawing the TOP VIEW and work from that to create the other four views. If you have trouble doing this, make a physical model; then use it to help you make your drawings. Practice doing this until you can make your drawings from only a top view.]

2. Each student must solve another student's Challenge Problem.
3. (Optional) Repeat Exercise 1 for a 4x4 array.
4. (Optional) Repeat Exercise 2 for a 4x4 array.

REPRESENTING VISUAL PERSPECTIVE VIA GRAPH PAPER
ANSWERS FOR LABORATORY EXAMPLE

FROM THE BACK VIEW

The first column shows an 8-story building in the facing row and nothing else is visible behind it. (This means that all the buildings behind the facing building have at most 8 stories.)

The second column shows a 4-story building in the facing row and a 9-story building behind it in one of the next two rows.

The third column shows a 7-story building in the facing row and nothing else is visible behind it. (This means that all the buildings behind the facing building are at most 7 stories.)

From the LEFT VIEW

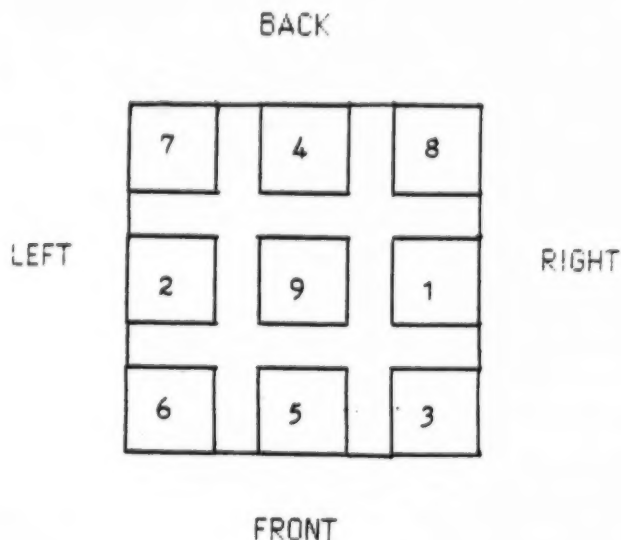
The first column shows a 7-story building in the facing row and an 8-story building behind it in one of the next two rows.

The second column shows a 2-story building in the facing row and a 9-story building behind it in one of the next two rows.

The third column shows a 6-story building in the facing row and nothing else visible behind it.

Solution for the Example:

The diagram shows the TOP VIEW containing building height information.



Note: Answers to the Laboratory Exercises (Challenge Problems) will vary.

REPRESENTING VISUAL PERSPECTIVE VIA GRAPH PAPER
HOMEWORK EXERCISES

1. Make a TOP VIEW diagram for any 4x4 array. Then make the diagrams for the four face views of the array.

2. (Optional) Make a TOP VIEW diagram for any 5x5 array. Then make the diagrams for the four face views of the array.

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: VISUAL PERSPECTIVES -- TEAM COMPETITION

OBJECTIVE:

To reinforce student skills in working with representations and interpretations of two- and three-dimensional views of objects.

COURSE: Grades 7,8; General Math 9-12; Geometry

TIME: one class period

SOFTWARE: Building Perspective

OTHER MATERIALS :

- Cuisenaire Rod Assortment
- 5" X 5" pieces of 1-inch styrofoam having holes (5 rows by 5 columns) that measure 1 cm by 1 cm by 1 cm
- or building clay (to use as the base instead of the styrofoam)
- or Multilinks cubes
- or Unifix cubes
- 3 X 3 Arrays Worksheet
- 4 X 4 Arrays Worksheet
- 5 X 5 Arrays Worksheet
- graph paper (4x4 per inch)

PREREQUISITE SKILLS/CONCEPTS:

The student should be able to

- use the software program
- identify the pieces in a set of Cuisenaire rods

FORMAT: laboratory

GENERAL DESCRIPTION OF THE LAB ACTIVITY:

A team contest format is used to help improve new skills and to add an element of enjoyment to the process.

VISUAL PERSPECTIVE -- TEAM COMPETITION
LABORATORY ACTIVITY
TEACHER'S GUIDE

This should be an enjoyable activity for all.

The directions are designed for a group competition, but they may be modified for use in a straightforward group activity. Also, they may be adapted for different ability levels by adjusting the sizes of the arrays required or the time allowances. (Time constraints could be dropped entirely.)

The software program should be loaded prior to the arrival of the students. Students may be arranged in teams of two, three, or four -- as convenient.

Be sure that all groups are synchronized with respect to their positions in the program before giving the signal to begin.

As shown in the directions, there is ordinarily no partial credit given for a round. This restriction may be modified for certain situations at teacher discretion.

Two matches may be run in a 45-minute period by adjusting the times to 3-, 5-, and 8-minute rounds respectively.

VISUAL PERSPECTIVE -- TEAM COMPETITION
LABORATORY ACTIVITY

Rules for the Competition

1. All teams will be working with the same size array at the same time.
2. Not all teams will be working with the same problem at the same time.
The computer program will choose the problems randomly.
3. When a team finishes a problem, THEY MUST CALL THE TEACHER TO MONITOR
THE CHECKING OF THEIR SOLUTION AGAINST THE PROGRAM'S ANSWER.
Failure to do this will result in disqualification for the round.
4. Upon completion of the problem(s) in a round, a team MUST NOT CONTINUE
ON TO THE NEXT ROUND UNTIL DIRECTED TO DO SO BY THE TEACHER. The penalty
for continuing is disqualification from the next round.
5. There shall be 3 Rounds in a Match.

Round 1 - Array: 3x3	Time limit: 5 min.	Points: 5
Round 2 - Array: 4x4	Time limit: 5 min.	Points: 5
Round 3 - Array: 5x5	Time limit: 10 min.	Points: 10

6. The winner is the team with the highest Match Score.

TITLE: THE GEOMETRIC SUPPOSER: TRIANGLES

PUBLISHER: Sunburst Communications
39 Washington Ave.
Pleasantville, N.Y. 10570

PRICE: \$99.00
10 - disk lab pack \$297.00

COMPUTERS: Apple IIc, IIe, IIgs

PIMMS RATING: Excellent

PROGRAM APPROPRIATE FOR:

Courses: Geometry, 7-12

Ability Level: low, middle, high

Format: class demo/instructional
discovery/laboratory
drill and practice

Characteristics: user friendly, documentation,
on-screen help, graphics,
edit functions, disk-independent

ADDITIONAL PERIPHERALS NEEDED: none

REVIEW AND DESCRIPTION OF THE PROGRAM:

The Geometric Supposer: Triangles permits the user to perform any construction that is possible with compasses and straightedge. Measurements can be made and scales changed to allow students to make conjectures about properties of and relationships among triangles. A REPEAT function allows the students to investigate their conjectures regarding a previous construction by repeating it with new figures of the users choice. [Students should be encouraged to make any conjectures that they feel might be valid.]

Students should be allowed to become familiar with the software by experimenting with the program options: Draw, Label, Erase, Measure, Scale, Repeat, New Triangle.

The Supposer can be an effective tool in motivating students to think about geometry by allowing them to discover theorems on their own without being given advance information. The teacher's role then is to provide meaningful demonstrations and investigations (which may set the stage for the later development of formal proofs, if desired).

The program documentation illustrates the use of each menu via simple examples, permitting the use of the program as a tutorial.

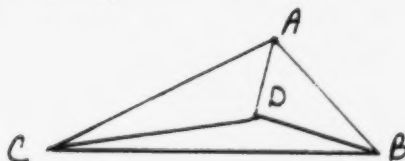
Other programs in the series include The Geometric preSupposer, The Geometric Supposer: Quadrilaterals, and The Geometric Supposer: Circles.

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

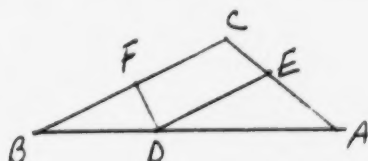
TOPIC: RANDOM POINTS OF TRIANGLES

OBJECTIVES:

To show that if a random point is chosen in the interior of a triangle, then the sum of the lengths from that point to the vertices of the triangle always must be greater than half the perimeter of the triangle.



To show that if a random point is chosen in one of the three sides of a triangle and connected to the midpoints of the other two sides of the triangle, then the area of the quadrilateral formed in the interior of the original triangle is equal to the sum of the areas of the two triangles which are formed in the interior of the original triangle.



COURSE: Geometry

SOFTWARE: GEOMETRIC SUPPOSER: TRIANGLES

TIME: one class period

PREREQUISITE SKILLS/CONCEPTS:

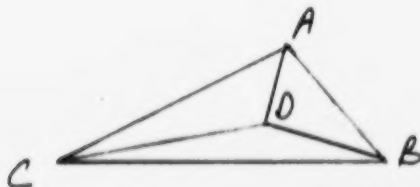
The student should be familiar with using the software program.

FORMAT: discovery lab

RANDOM POINTS OF TRIANGLES LABORATORY EXERCISES

EXERCISE 1

Use the Supposer program. Follow the directions below to LOCATE A RANDOM POINT IN THE INTERIOR OF A TRIANGLE. The following diagram shows the figure that the directions will enable you to construct. The random point is given by D in the diagram.



DIRECTIONS: Follow the sequence as you read down the list.
Note: ESCape may be used to go back to a previous response.

MENU CHOICE	RESPONSE
-----	-----
New Shape	N
Type	2
Label	2
Random Point	4
Inside Triangle	2
	ABC
Draw	1
Segment	1
	AD
Draw	1
Segment	1
	CD
Draw	1
Segment	1
	BD

RANDOM POINTS OF TRIANGLES
LABORATORY EXERCISES

EXERCISE 2

Use the Supposer program. Construct triangle ABC having random point D in the interior of the triangle. Connect D to A,B, and C.

Select the measure option and choose 2: Perimeter. Find and compare the sum of lengths AD, BD, CD with the perimeter of triangle ABC, then form a conjecture about this relationship.

Next, use the REPEAT function with the following types of triangles to help support your conjecture:

a) right b) acute c) obtuse d) isosceles e) equilateral.

MEASUREMENTS

Triangle ABC	AD	BD	CD	Perimeter ABC
--------------	----	----	----	---------------

a) Right

b) Acute

c) Obtuse

d) Isosceles

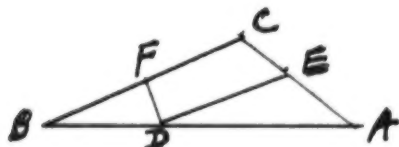
e) Equilateral

f) Use all your data. Formulate and state your conjecture.

RANDOM POINTS OF TRIANGLES
LABORATORY EXERCISES

EXERCISE 3

Use the Supposer program. Follow the directions below to LOCATE A RANDOM POINT IN ONE OF THE SIDES OF A TRIANGLE AND THE MIDPOINTS IN THE OTHER TWO SIDES. The following diagram shows the figure that the directions will enable you to draw.



DIRECTIONS: Follow the sequence as you read down the list.
Note: ESCape may be used to go back to a previous response.

MENU CHOICE	RESPONSE
-----	-----
New Shape	N
Type	2
Label	2
Random Point	4
On Segment	1
	AB
Label	2
Subdivide Segment	2
	AC
Number of Sections	2
Label	2
Subdivide Segment	2
	BC
Number of Sections	2
Draw	1
Segment	1
	DF
Draw	1
Segment	1
	DE

RANDOM POINTS OF TRIANGLES LABORATORY EXERCISES

EXERCISE 4

Use the Supposer program. Construct triangle ABC with random point D in segment AB.

Use the label/subdivide option. Locate the midpoints of segments AC and BC. (The computer will label them E and F respectively.)

Draw segments DE and DF.

Compare the areas of the two triangles formed with the area of the quadrilateral*.

State a conjecture about this relationship.

Next, use the REPEAT function with the following types of triangles to help support your conjecture:

a) right b) acute c) obtuse d) isosceles e) equilateral.

* Note: To find the area of quadrilateral CEDF always find the area of triangle EDF followed by " + " and then the area of triangle ECF.

MEASUREMENTS

	Triangle ADE	Triangle BDF	Quadrilateral CEDF
--	--------------	--------------	--------------------

a) Right

b) Acute

c) Obtuse

d) Isosceles

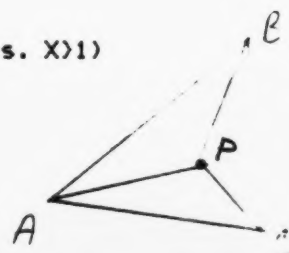
e) Equilateral

f) Use all your data. Formulate and state your conjecture.

RANDOM POINTS OF TRIANGLES HOMEWORK EXERCISES

1. Use the diagram at the right. Find X. (Give X in the simplest form of an inequality. Present results as a completed table.)

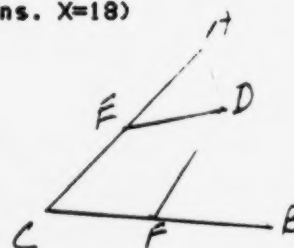
	AB	BC	AC	PA	PB	PC
Example: 3	4	5	2	3	X	(ans. $X > 1$)
a)	6	4	8	3	4	X
b)	8	10	12	6	X	7
c)	8	7	X	6.5	3.5	5
d)	X	13	10	3	X	6



ex. 1

2. Show that no matter how P is chosen inside triangle ABC, it will always be true that
 $(PA + PB + PC) > .5(AB + BC + AC)$.
 (Hint: Use the triangle inequality.)
3. Given triangle ABC with random point D in segment AB and midpoints E and F in segments AC and BC, find X. Present your results as a completed table.

	Area of ADE	Area of DBF	Area of CEDF
Example: X	12	30	(ans. $X=18$)
a)	16	X	36
b)	$X + 6$	X	20
c)	$X - 6$	X	$X + 8$
d)	$1.5(X - 2)$	$X + 3$	$2X + 9$

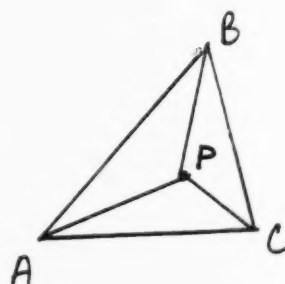


ex. 3

RANDOM POINTS OF TRIANGLES
ANSWERS TO HOMEWORK EXERCISES

1. Use the diagram at the right. Find X. (Give X in the simplest form of an inequality. Present results as a completed table.)

	AB	BC	AC	PA	PB	PC
Example:	3	4	5	2	3	$x > 1$
a)	6	4	8	3	4	$x > 2$
b)	8	10	12	6	$x > 2$	7
c)	8	7	$x < 15$	6.5	3.5	5
d)	$x > 5$	13	10	3	x	6



ex. 1

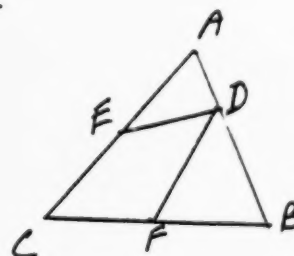
2. Show that no matter how P is chosen inside triangle ABC, it will always be true that $(PA + PB + PC) > .5(AB + BC + AC)$. (Hint: Use the triangle inequality.)

$$\begin{aligned} PA + PB &> AB \\ PB + PC &> BC \\ PC + PA &> AC \end{aligned}$$

By summation, $2(PA + PB + PC) > AB + BC + AC$,
or, $PA + PB + PC > .5(AB + BC + AC)$.

3. Given triangle ABC with random point D in segment AB and midpoints E and F in segments AC and BC, find X. Present your results as a completed table.

	Area of ADE	Area of DBF	Area of CEDF
Example:	18	12	30
a)	16	$x = 20$	36
b)	13	$x = 7$	20
c)	8	$x = 14$	22
d)	24	$x = 21$	45



ex. 3

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

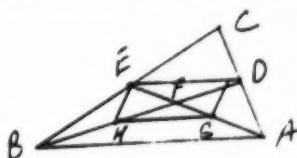
TOPIC: MEDIANS OF A TRIANGLE

OBJECTIVES:

To show that the medians of a triangle intersect in one point.

To show that intersecting medians of a triangle divide each other in the ratio 2:1.

To show that when two medians are divided into three equal parts, a parallelogram may be constructed in a triangle.



COURSE: Geometry

SOFTWARE: GEOMETRIC SUPPOSER: TRIANGLES

TIME: one class period

PREREQUISITE SKILLS/CONCEPTS:

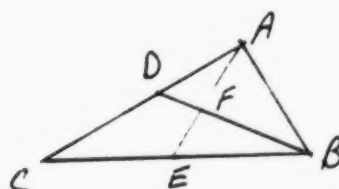
Student should be familiar with using the software.

FORMAT: discovery lab

MEDIANS OF A TRIANGLE LABORATORY EXERCISES

EXERCISE 1

Use the Supposer program. Follow the directions below to CONSTRUCT TWO MEDIANS OF A TRIANGLE. The following diagram shows the figure that the directions will enable you to construct.



DIRECTIONS: Follow the sequence as you read down the list.
Note: EScAPE may be used to go back to a previous response.

MENU CHOICE	RESPONSE
-----	-----
New Shape	N
Type	3
Rescale	S
Draw	1
Median	3
In Triangle	ABC
From Vertex	B
Draw	1
Median	3
In Triangle	ABC
From Vertex	A
Label	2
Intersection	1
Of line	AE
With line	BD

MEDIANS OF A TRIANGLE LABORATORY EXERCISES

EXERCISE 2

Use the Geometric Supposer. Construct triangle ABC with medians BD and AE intersecting at point F.

Use the MEASURE option. Compare DF with BD. Compare EF with AE and form a conjecture about this relationship.

Next, use the REPEAT function with other types of triangles to help support your conjecture.

MEASUREMENTS

Triangle ABC	DF	BD	EF	AE
--------------	----	----	----	----

a) Right

b) Acute

c) Obtuse

d) Isosceles

e) Equilateral

f) Use all your data. Formulate and state your conjecture.

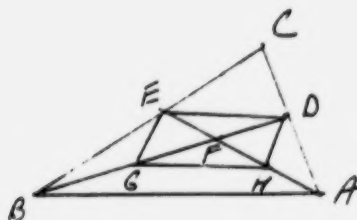
g) Use the Supposer. Draw the median from vertex C. What appears to be true about the intersection of the third median with the other two?

Repeat this construction on another type of triangle to help support your conjecture.

MEDIANS OF A TRIANGLE LABORATORY EXERCISES

EXERCISE 3

Use the Supposer program. Follow the directions below to STUDY FURTHER PROPERTIES ABOUT MEDIANS. The following diagram shows the figure that the directions will enable you to construct.



DIRECTIONS: Use instructions from previous exercises in addition to those below. Follow the sequence as you read down the list.
Note: EEscape may be used to go back to a previous response.

MENU CHOICE	RESPONSE
-----	-----
Label	2
Subdivide Segment	2
Segment	BF
Number of Sections	2
Label	2
Subdivide Segment	2
Segment	AF
Number of Sections	2
Draw	1
Segment	1
	DE
Draw	1
Segment	1
	EG
Draw	1
Segment	1
	GH
Draw	1
Segment	1
	DH

MEDIANS OF A TRIANGLE
LABORATORY EXERCISES

EXERCISE 4

Use the Supposer program. Construct triangle ABC with medians BD and AE intersecting at F. Use the program directions to locate points G and H in medians BD and AE. Compare DE to GH and EG to DH, then state a conjecture about such a relationship. Use the REPEAT function and other types of triangles to help support your conjecture.

- a) Draw a figure and identify the type of triangle which you have chosen.

Triangle 1

DE =

GH =

EG =

DH =

Use your data. State a conjecture about figure DEGH.

Repeat the same construction on another type of triangle.

- b) Draw a figure and identify the type of triangle which you have chosen.

Triangle 2

DE =

GH =

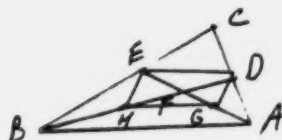
EG =

DH =

State your conclusion(s) from these activities.

MEDIANS OF A TRIANGLE
HOMEWORK EXERCISES

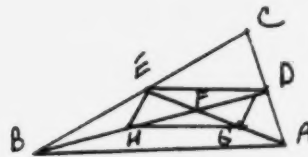
Consider triangle ABC with medians $\overline{AE}$ and $\overline{BD}$, F the intersection of $\overline{AE}$ and $\overline{BD}$, and G and H the midpoints of $\overline{AF}$ and $\overline{BF}$ respectively ...



- 1) If $AE = 9$, then $FE =$, $AF =$
- 2) If $DF = 2.5$, then $BF =$, $BD =$
- 3) If $FE = X + 3$, and $AE = 6X + 3$, then $AE =$, $FE =$
- 4) If $BD = X + 9$, and $FD = 3X - 1$, then $BD =$, $FD =$
- 5) If $DE = 8$, and $DG = 3$, then $EH =$, and $GH =$
- 6) If $GH = 2X + 3$ and $DE = 4X - 6$ then $GH =$, $DE =$
- 7) $AD = 3X$, and $DC = 2X + 4$, $AD =$, $DC =$
- 8) $BC = 8X - 3$, $EC = 3X + 1$, $BC =$, $EC =$
- 9) Name two pairs of congruent triangles in parallelogram $DEGH$.
- 10) State the relationship of GF to FE and DF to FH . Explain this relationship.

MEDIANS OF A TRIANGLE
ANSWERS TO HOMEWORK EXERCISES

Consider triangle ABC with medians $\overline{AE}$ and $\overline{BD}$, F the intersection of $\overline{AE}$ and $\overline{BD}$, and G and H the midpoints of $\overline{AF}$ and $\overline{BF}$ respectively ...



- 1) If $AE = 9$, then $FE = 3$, $AF = 6$.
- 2) If $DF = 2.5$, then $BF = 5$, $BD = 7.5$.
- 3) If $FE = X + 3$, and $AE = 6X + 3$, then $AE = 15$, $FE = 5$.
- 4) If $BD = X + 9$, and $FD = 3X - 1$, then $BD = 10.5$, $FD = 3.5$.
- 5) If $DE = 8$, and $DG = 3$, then $EH = 3$, and $GH = 8$.
- 6) If $GH = 2X + 3$ and $DE = 4X - 6$ then $GH = 12$, $DE = 12$.
- 7) $AD = 3X$, and $DC = 2X + 4$, $AD = 12$, $DC = 12$.
- 8) $BC = 8X - 3$, $EC = 3X + 1$, $BC = 17$, $EC = 8.5$.
- 9) Name two pairs of congruent triangles in parallelogram $DEGH$.

 DEF and HFG , DFG and HFE
- 10) State the relationship of GF to FE and DF to FH .
 Explain this relationship.

 The ratios are equal because the diagonals of a parallelogram bisect each other.

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: HERON'S FORMULA

OBJECTIVES:

To show that if the lengths of sides a, b, and c of a triangle are known then the area (K) of the triangle is given by Heron's Formula: $K = \sqrt{s(s-a)(s-b)(s-c)}$.
[Here, s represents the semiperimeter of the triangle:
 $s = .5(a+b+c)$.]

COURSE: Geometry

SOFTWARE: THE GEOMETRIC SUPPOSER: TRIANGLES

TIME: one class period

PREREQUISITE SKILLS/CONCEPTS:

The student should be familiar with the software.

FORMAT: discovery lab

HERON'S FORMULA LABORATORY EXERCISES

EXERCISE 1

Use the Geometric Supposer:Triangles. Construct right triangle ABC. Draw an altitude from vertex A. Use the MEASURE option to determine the lengths of the altitude and base (Ex.1a,b). Compute the area of ABC using $K = .5 \times \text{altitude} \times \text{base}$ (Ex.1c). Then MEASURE AB,BC,and AC (Ex.1d - f). Use Heron's Formula to compute the area (Ex.1g), and state your conjecture about the results (Ex.1h).

RIGHT TRIANGLE

a) Altitude

b) Base

c) Area=.5ab

d) AB

e) BC

f) AC

g) Use a calculator. Complete the indicated computations.

Find s

Find K

$$s = .5 (\text{Perimeter})$$

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = .5(a + b + c)$$

=

$$s = .5(\quad + \quad + \quad)$$

=

$$s = .5(\quad)$$

=

$$s =$$

=

h) Compare the areas found by the two methods. State a conjecture based on your data and computations.

EXERCISE 2

Use the Repeat function. Construct the following types of triangles. Record data and carry out computations as in Ex. 1.

ACUTE

OBTUSE

a) Altitude

b) Base

c) Area = .5ab

d) AB

e) BC

f) AC

g) Use a calculator. Complete the indicated computations.

Find s

Find K

$$s = .5 (\text{Perimeter})$$

$$s = .5(a + b + c)$$

$$s = .5(\quad + \quad + \quad)$$

$$s = .5(\quad)$$

$$s =$$

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

$$=$$

$$=$$

$$=$$

$$=$$

ISOSCELES

EQUILATERAL

h) Altitude

j) Base

k) Area = .5ab

m) AB

n) BC

p) AC

q) Use a calculator. Complete the indicated computations.

Find s

Find K

$$s = .5 (\text{Perimeter})$$

$$s = .5(a + b + c)$$

$$s = .5(\quad + \quad + \quad)$$

$$s = .5(\quad)$$

$$s =$$

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

$$=$$

$$=$$

$$=$$

$$=$$

r) What conclusion can we make about finding the area of any triangle if the sides are given?

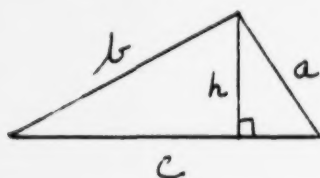
HERON'S FORMULA HOMEWORK EXERCISES

Given triangle ABC with sides a , b , and c , compute s .
[semiperimeter: $s = .5(a + b + c)$]. Record results in the table
below. Give areas in simplest radical form and in decimal form
rounded off to tenths. Use a calculator.

TRIANGLE ABC					
	a	b	c	s	Area
1)	3	4	5		
2)	4	5	6		
3)	9	11	13		
4)	7	8	9		
5)	8	10	12		

EXTENSION:

6) Use the figure as labeled below. Prove Heron's Formula holds.



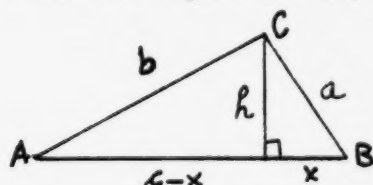
HERON'S FORMULA ANSWERS TO HOMEWORK EXERCISES

Given triangle ABC with sides a, b, and c, compute s.
[semiperimeter: $s = .5(a + b + c)$]. Record results in the table
below giving areas in simplest radical form and in decimal form
rounded off to tenths. Use a calculator.

TRIANGLE ABC					
	a	b	c	s	Area
1)	3	4	5	6	6
2)	4	5	6	7.5	$\frac{15\sqrt{7}}{4}$, or 9.9
3)	9	11	13	16.5	$\frac{33\sqrt{35}}{4}$, or 48.8
4)	7	8	9	12	$12\sqrt{5}$, or 26.8
5)	8	10	12	15	$15\sqrt{7}$, or 39.7

EXTENSION:

6) Use the figure as labeled below. Prove Heron's Formula holds.



$$K_{ABC} = \frac{1}{2}ch$$

$$K_{ABC}^2 = \frac{1}{4}c^2h^2 = \frac{1}{4}c^2(a^2 - x^2) = \frac{1}{4}c^2(a-x)(a+x)$$

$$= \frac{1}{4}c^2 \left[\frac{b^2 - (a-c)^2}{2c} \right] \left[\frac{(a+c)^2 - b^2}{2c} \right]$$

$$= \frac{1}{4}c^2 \left[\frac{(b-a+c)(b+a-c)}{2c} \right] \left[\frac{(a+c-b)(a+c+b)}{2c} \right]$$

$$= \frac{1}{4}c^2 \left[\frac{4(s-a)(s-b)}{2c} \right] \left[\frac{4s(s-c)}{2c} \right]$$

$$K_{ABC}^2 = s(s-a)(s-b)(s-c)$$

$$K_{ABC} = \sqrt{s(s-a)(s-b)(s-c)}$$

Express a+x and a-x

By Pyth. Thm., $a^2 - x^2 = b^2 - (c-x)^2$

therefore, $x = \frac{a^2 + c^2 - b^2}{2c}$

and $a-x = \frac{b^2 - (a-c)^2}{2c}$

and $a+x = \frac{(a+c)^2 - b^2}{2c}$

Express a, b, c using s

$$s = \frac{1}{2}(a + b + c)$$

thus, $a = 2s - b - c$

and $b = 2s - a - c$

and $c = 2s - a - b$

TITLE: THE GEOMETRIC SUPPOSER: QUADRILATERALS

PUBLISHER: Sunburst Communications
39 Washington Ave.
Pleasantville, N.Y. 10570

PRICE: \$99.00
10 - disk lab pack \$297.00

COMPUTERS: Apple +, IIc, IIe, IIgs

PIMMS RATING: Excellent

PROGRAM APPROPRIATE FOR:

Courses: Geometry, 7-12

Ability Level: low, middle, high

Format: class demo/instructional
discovery/laboratory,
drill and practice

Characteristics: user friendly, documentation,
on-screen help, graphics,
edit functions, disk-independent

ADDITIONAL PERIPHERALS NEEDED: none

REVIEW AND DESCRIPTION OF THE PROGRAM:

The Geometric Supposer: Quadrilaterals is one of four software packages that permit the user to explore the properties of geometric figures and make conjectures about those properties.

The program allows the user to perform any construction that is possible with compasses and straightedge. Measurements can be made of angles, lengths, areas, perimeters, distance from a point to a line, and distance between two lines. Change of scale also is permitted. Students then may make conjectures about properties of and relationships among quadrilaterals. A REPEAT function allows the students to investigate their conjectures regarding a previous construction by repeating it with new figures of their own choice. [Students should be encouraged to make any conjectures that they feel might be valid.]

Students should be allowed to become familiar with the software via experimentation with the program options: Draw, Label, Erase, Measure, Scale, Repeat, New Triangle.

The Supposer can be an effective tool in motivating students to think about geometry by allowing them to discover theorems on their own. The teacher's role then is to provide meaningful demonstrations and investigations (which may set the stage for the later development of formal proofs, if desired).

The program documentation illustrates the use of each menu via simple examples, permitting the use of the program as a tutorial.

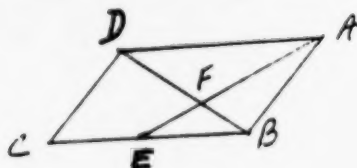
Other programs in the series include The Geometric preSupposer, The Geometric Supposer: Triangles, and The Geometric Supposer: Circles.

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: RATIOS OF AREAS WITHIN A PARALLELOGRAM

OBJECTIVE:

To show that if a segment is drawn from one vertex of a parallelogram to the midpoint of an opposite side and if the diagonal is drawn between the other two vertices, then the areas of the four regions formed have ratios 1:2:4:5.



COURSE: Geometry

SOFTWARE: THE GEOMETRIC SUPPOSER: QUADRILATERALS

TIME: one class period

PREREQUISITE SKILLS/CONCEPTS:

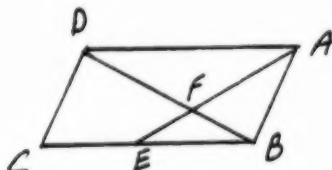
Student should : be familiar with using the software
be able to identify similar triangles
be able to work with ratios

FORMAT: discovery lab

AREAS WITHIN A PARALLELOGRAM LABORATORY EXERCISES

EXERCISE 1

Use the Geometric Supposer: Quadrilaterals program. Follow the directions below to DRAW A SEGMENT JOINING A VERTEX OF A PARALLELOGRAM TO THE MIDPOINT OF AN OPPOSITE SIDE AND THE DIAGONAL BETWEEN THE OTHER TWO VERTICES. The following diagram shows the figure that the directions will enable you to construct.



DIRECTIONS: Follow the sequence as you read down the list.
Note: ESCape may be used to go back to a previous response.

MENU CHOICE	RESPONSE
-----	-----
New Shape	N
Parallelogram	1
Type	1
Scale	S
Label	2
Subdivide Segment	2
Subdivided Segment	BC
Number of Sections	2
Draw	1
Segment	1
Segment Name	AE
Draw	1
Segment	1
Segment Name	BD
Label	2
Intersection	1
Intersection of Line	AE
With Line	BD

AREAS WITHIN A PARALLELOGRAM
LABORATORY EXERCISES

EXERCISE 2

Use the Geometric Supposer: Quadrilaterals program. Construct a random parallelogram, ABCD. (Done as Exercise 1.)

Use the LABEL/SUBDIVIDE option: Use E for the midpoint of $\overline{BC}$. Draw segment AE and diagonal BD. Use F for the intersection of $\overline{AE}$ and $\overline{BD}$. Find the areas of the four regions formed (Ex. 1a - 1d).

Next, find the ratios among these areas (Ex. 1e - 1g), and make a conjecture about this relationship (Ex. 1h).

PARALLELOGRAM 1 (Random)

Find the area.

- a) Area of BEF =
- b) Area of ABF =
- c) Area of ADF =
- d) Area of DCEF =

Find the ratio.

e)
$$\frac{\text{Area of ABF}}{\text{Area of BEF}} = \frac{\text{-----}}{\text{-----}} =$$

f)
$$\frac{\text{Area of ADF}}{\text{Area of BEF}} = \frac{\text{-----}}{\text{-----}} =$$

g)
$$\frac{\text{Area of DCEF}}{\text{Area of BEF}} = \frac{\text{-----}}{\text{-----}} =$$

h) State a conjecture about the following ratios:

Area of BEF : Area of ABF : Area of ADF : Area of DCEF

EXERCISE 3

Use the REPEAT function. Analyze the same relationships with a rectangle, a rhombus, and a square.

PARALLELOGRAM ABCD

MEASUREMENTS	RECTANGLE	RHOMBUS	SQUARE
a) Area of BEF			
b) Area of ABF			
c) Area of ADF			
d) Area of DCEF			
e)			
Area of ABF			
Area of BEF			
f)			
Area of ADF			
Area of BEF			
g)			
Area of DCEF			
Area of BEF			
h) Does the above data support the conjecture made in Ex.2?			
i) Would the conjecture be supported by data from ANY quadrilateral? Why?			

a) Area of BEF

b) Area of ABF

c) Area of ADF

d) Area of DCEF

e)

Area of ABF

Area of BEF

f)

Area of ADF

Area of BEF

g)

Area of DCEF

Area of BEF

h) Does the above data support the conjecture made in Ex.2?

i) Would the conjecture be supported by data from ANY quadrilateral? Why?

AREAS WITHIN A PARALLELOGRAM HOMEWORK EXERCISES

Consider a parallelogram having the following properties:

$\overline{BD}$ is a diagonal

E is the midpoint of $\overline{DC}$

$\overline{AE}$ is a segment from vertex A to the midpoint of $\overline{DC}$

F is the intersection of $\overline{BD}$ and $\overline{AE}$



Compute the areas indicated in the table. (See the next page for two worked examples of how to do this.)

AREA of	DEF	ABF	ADF	BCEF	ABCD
1)	6				
2)					48
3)			9		
4)				1.2	
5)		11			
6)			$x+3$		$4x$
7)		$2x+5$		$8x$	
8)				$3x-1$	$5x+2$
9)	$.5x+4$		$5x+1$		

EXTENSION:

- 10) If a segment is drawn from one vertex to the midpoint of an opposite side and a diagonal is drawn between the other vertices as shown in the diagram, show how we can find the areas of the four regions (I, II, III, IV).



AREAS WITHIN A PARALLELOGRAM HOMEWORK EXAMPLES

EXAMPLE 1

AREA of	DEF	ABF	ADF	BCEF	ABCD
	$\wedge$ 	$\wedge$ 	$\wedge$ 	$\wedge$ 	$\downarrow$
Use	x for	4x for	2x for	5x for	60

\Find x\

$$\begin{array}{rclclclclcl}
 x & + & 4x & + & 2x & + & 5x & = & 60 \\
 & & & & & & 12x & = & 60 \\
 & & & & & & x & = & 5
 \end{array}$$

Conclusion: 5 20 10 25 60

EXAMPLE 2

Use $\wedge$
| $\wedge$
|
 $x+6$ for area ADF, and $5x$ for area ABCD.

\Find x\

$$\frac{\begin{array}{c} K \\ ADF \end{array}}{\begin{array}{c} K \\ ABCD \end{array}} = \frac{4}{12}$$

$$\frac{x+6}{5x} = \frac{1}{3}$$

$$x = 9$$

Conclusion: 9 36 18 45 108

AREAS WITHIN A PARALLELOGRAM
ANSWERS TO HOMEWORK EXERCISES

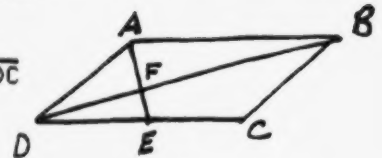
Consider a parallelogram having the following properties:

$\overline{BD}$ is a diagonal

E is the midpoint of $\overline{DC}$

$\overline{AE}$ is a segment from vertex A to the midpoint of $\overline{DC}$

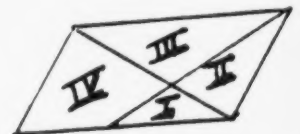
F is the intersection of $\overline{BD}$ and $\overline{AE}$



Compute the areas indicated in the table.

AREA of	DEF	ABF	ADF	BCEF	ABCD
1)	6	12	24	30	72
2)	4	8	16	20	48
3)	2.25	4.5	9	11.25	27
4)	.24	.48	.96	1.2	2.88
5)	5.5	11	22	27.5	66
6)	3	6	12	15	36
7)	20/3	40/3	80/3	100/3	80
8)	1	2	4	5	12
9)	6.5	13	26	32.5	78

10)



Similar Δ .
Sides: 2:1
Areas: 4:1



Bases: 2:1
Alts are =
Areas: 2:1

$$\Rightarrow K_I : K_{II} : K_{III} = 1 : 2 : 4$$

Find K_{IV}

$$K_{IV} + K_I = K_{III} + K_{II}$$

$$\begin{aligned} K_{IV} &= K_{III} + K_{II} - K_I \\ &= 4 + 2 - 1 \\ &= 5 \end{aligned}$$

Thus $K_I : K_{II} : K_{III} : K_{IV} = 1 : 2 : 4 : 5$

COMPUTER SOFTWARE ACTIVITY LESSON PLAN

TOPIC: AREAS FORMED BY
DIAGONALS OF PARALLELOGRAMS AND TRAPEZOIDS

OBJECTIVES:

To show that:

the two diagonals of a parallelogram (or of a trapezoid) give two pairs of triangles having equal areas.

when the diagonals of a parallelogram (or trapezoid) are drawn, the product of the areas of two nonadjacent angles formed are equal to the product of the areas of the other two non adjacent triangles.

COURSE: Geometry

SOFTWARE: THE GEOMETRIC SUPPOSER: QUADRILATERALS

TIME: one class period

PREREQUISITE SKILLS/CONCEPTS:

The student should be familiar with the software.

FORMAT: discovery lab

AREAS AND DIAGONALS OF QUADRILATERALS LABORATORY EXERCISES

EXERCISE 1

Use the Geometric Supposer: Quadrilaterals. Construct parallelogram ABCD with diagonals AC and BD intersecting at E. (Use the LABELS option.). Find the areas described in the table below (Ex.1a - 1f). Then state conjectures based on your data (Ex.1g,h).



PARALLELOGRAM ABCD

	RANDOM	RECTANGLE	RHOMBUS	SQUARE
a) AREA ACD				
b) AREA BCD				
c) AREA AED				
d) AREA BEC				
e) AREA AEB				
f) AREA DEC				
g) Use your data. State conjectures about the areas of AED and BEC: ACD and BCD: Why does this seem to be true?				
h) What conjecture can be made about the product of areas AED and CEB? areas DEC and AEB?				

AREAS AND DIAGONALS OF QUADRILATERALS LABORATORY EXERCISES

EXERCISE 2

Use the REPEAT function. Produce the same construction on different types of trapezoids. Make conjectures about the same area relationships as in Exercise 1. Then compare the results for trapezoids with those for parallelograms.

TRAPEZOID ABCD

	RANDOM	ISOSCELES	RIGHT ANGLE
--	--------	-----------	-------------

a) AREA ACD

b) AREA BCD

c) AREA AED

d) AREA BEC

e) AREA AEB

f) AREA DEC

g) State conjectures about the areas of:

AED and BEC

ACD and BCD

h) Make a conjecture about the products of areas AED and CEB and areas DEC and AEB.

i) How do the results of your investigation of trapezoids differ from those for parallelograms? How are they the same?

AREAS AND DIAGONALS OF QUADRILATERALS LABORATORY EXERCISES

EXERCISE 3

Use the REPEAT function. Select the YOUR OWN option.
Produce the same construction on a Kite and a quadrilateral.
Find the same kind of data (Ex. 3 a - f), and make
conjectures about the same relationships (Exs.3 g - h).
Then compare the results for ANY QUADRILATERAL (yours) with
those for parallelograms (Ex. 3j).

QUADRILATERAL ABCD

KITE	MY OWN QUADRILATERAL
------	----------------------

a) AREA ACD

b) AREA BCD

c) AREA AED

d) AREA BEC

e) AREA AEB

f) AREA DEC

g) Does the above data support your conjectures about
the areas of AED and BEC?

the areas of ACD and BCD?

h) Is the conjecture made about the respective products of
areas AED and CEB and DEC and AEB the same?

j) Which conjecture about areas must be modified for the
above quadrilaterals? Which conjecture is the same?

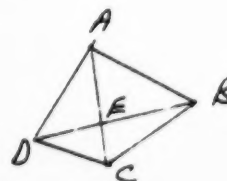
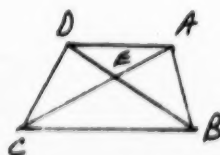
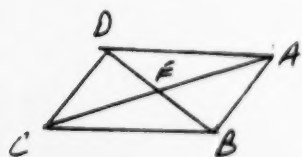
AREAS AND DIAGONALS OF QUADRILATERALS HOMEWORK EXERCISES

Use the results of the Laboratory Exercises. Complete the table below by giving a YES or NO response in each "block".

AREA	AED = BEC ?	ACD = BCD ?	AED * CEB = DEC * AEB ?
1) Parallelogram			
2) Trapezoid			
3) Kite			
4) Own			

For exercises 5-12, diagonals AC and BD of quadrilateral ABCD intersect at E.

AREA of	AED	CEB	DEC	AEB
5) Parallelogram	4	7	2	X
6) Parallelogram	3	X+2	4	X-1
7) Trapezoid	3	6	X	5
8) Trapezoid	3	X	6	X-4
9) Kite	5	6	4	X
10) Kite	X+4	4	5	X+3
11) Random Quad	4	9	12	X
12) Random Quad	X-1	6	X+4	5



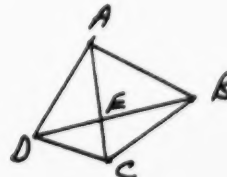
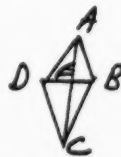
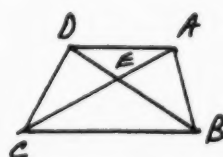
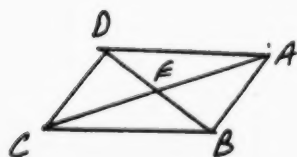
AREAS AND DIAGONALS OF QUADRILATERALS ANSWERS TO HOMEWORK EXERCISES

Use the results from the Laboratory Exercises. Complete the table below by giving a YES or NO response in each "block".

AREA	AED = BEC ?	ACD = BCD?	AED * CEB = DEC * AEB ?
1) Parallelogram	yes	yes	yes
2) Trapezoid	yes	yes	yes
3) Kite	no	no	yes
4) Own	no	no	yes

For exercises 5-12 given quadrilateral ABCD has diagonals AC and BD intersecting at E.

AREA of	AED	CEB	DEC	AEB
5) Parallelogram	4	7	2	14
6) Parallelogram	3	12	4	9
7) Trapezoid	3	6	18/5	5
8) Trapezoid	3	8	6	4
9) Kite	5	6	4	7.5
10) Kite	5	4	5	4
11) Random Quad	4	9	8	3
12) Random Quad	25	6	30	5





Computer Software Demonstration

A pimms MATHEMATICS AWARENESS WORKSHOP

Place:

Date:

Target audience: Secondary mathematics teachers and department heads **Maximum:** 30

Presenters: Don Hastings, Stratford H.S.; Geo. Parker, E.O. Smith H.S., Storrs

Content and Purpose: Computer technology is having a great impact upon mathematics education. For example, the rapid generation of images can be used to convey concepts visually and thereby provide more classroom time for analysis and conjecture-testing. In this workshop we will demonstrate the use of exemplary commercial software that can enhance teaching towards specific mathematical objectives. The software packages include the Geometric preSupposer, Supposer, Superplot, Green Globes/Graphing Equations, and selected programs from the MECC series. Programs dealing with spatial visualization and general math also will be demonstrated. Computer ethics and classroom management will be addressed. All software currently used is Apple-based.

Time: Refreshments will be available at 3:30 p.m. The workshop runs from 4 p.m. to 8 p.m. with a break for a meal at about 6 p.m.

Cost: A registration fee of \$ 33 contributes to the direct costs of the workshop.
(approx.) In addition, \$ 5 of printed materials will be issued. Please make the check in the amount of \$ 38 per participant payable to PIMMS.

Contact: Send your check and registration form **FOR OUR RECEIPT BY**
to
(varies with location of the workshop)

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Districts or Regional Centers wishing to host a Workshop should contact the pimms office (347-9411 x2688). Desirable hardware for mounting a workshop: 1 computer per two participants, 1 printer with graphics capability, at least one color monitor; overhead projector with screen, small chalkboard.

pimms Curriculum Development Publications

- * Apple Microcomputer Software Evaluation, August 1985; 14 pp. (free)
- * A Review of Selected Microcomputer Software Packages with Lessons for Teaching Mathematics, Grades 8 - 12, Vol. 1, April 1987; 122 pp. \$4.50
- * A Review of Selected Microcomputer Software Packages with Lessons for Teaching Mathematics, Grades 8 - 12, Vol. 2, April 1988; 170 pp. \$6
- * A Review of Selected Microcomputer Software Packages with Lessons for Teaching Mathematics, Grades 8 - 12, Vol. 3, **April 1989**; 137 pp. \$6
- * A Guide to Curriculum Development in Applied Mathematics: A Three-year Sequence for Noncollege Bound Students (Vol. 1 of 3, preliminary edition), Fall 1986; 82 pp. \$3.25
- * Down to Earth Chemistry, (for the non-college-bound), 3 sample units, February 1987; 60 pp. (free)
- * Down to Earth Chemistry, (for the non-college-bound), April 1988; 337 pp. \$10
- * A Guide to Computer Use by the Science Teacher, October 1987 c.; 70 pp. \$3
- * Microcomputer Activities for the Teaching of Science and Thinking Skills in the Elementary School, Grades 3 - 7, April 1988; 42 pp. \$3